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A Hetero-functional Graph Theory for Modeling Interdependent Smart City Infrastructure

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From Wester to my sister Vivien

From Inas & Amro to our daughters
Amina & Ayah

Preface

Cities have always played a prominent role in the prosperity of civilization. Indeed, every great civilization we can think of is associated with the prominence of one or more thriving cities. And so understanding cities – their inhabitants, their institutions, their infrastructure – what they are and how they work independently and together – is of fundamental importance to our collective growth as a human civilization. But the 21^{st} century city is different. No longer do they primarily exist as urban islands in a sea of rural life but they are now ever-more connected with other cities around the world in a network of urbanized population centers. The rise of globalization as an economic activity has brought about a globalized metropolitan culture to the point where many city dwellers are more likely to hop from one city to another than to explore the wilderness of rural life. The 21^{st} century is also giving rise to ***mega-cities*** with more than 10 million inhabitants each. While more than 40 cities worldwide hold this distinction at present, population growth and urbanization are set to create many more. And of course, the 21^{st} century is infused with ubiquitous technology that fundamentally affects how society, political and economic institutions, and infrastructure develop and function. Perhaps such ***smart cities*** are best viewed as the ultimate ***engineering system***: A class of systems characterized by a high degree of technical complexity, social intricacy, and elaborate processes aimed at fulfilling important functions in society.

Why this book?

With all of these fundamental changes in the nature of cities in the 21^{st} century, it is only reasonable to ask if our collective knowledge of engineering, science, and social science is enough to understand how this new type of complex engineering system works. To truly appreciate an answer to this question, we have to understand how we got here.

The 19^{th} and 20^{th} centuries were marked with new and unprecedented inventions like the generator, the telephone, and the automobile. These technologies developed simultaneously along two fundamentally orthogonal trajectories. In the ***reductionist*** direction, these technologies developed as individual products with ever greater speed and precision. Many such devices grew in size to achieve economies of scale and others were miniaturized to even the nanometer length scale. Our knowledge of the ***physical sciences*** grew accordingly to meet the needs of innovation in technological marketplaces. In the ***integrationist*** direction, these individual technology products were connected to form many of the large-scale infrastructure networks we know today: the power grid, the communication infrastructure, and the transportation system. Over time, these networked systems developed even more interactions while continuing to incorporate many new technologies like solar panels, smart phones, and electric vehicles. Consequently, our knowledge of the ***informatic sciences*** developed to keep track of the tremendous information required to represent these engineering systems of ever greater scope and function.

Two such informatic sciences are particularly relevant here. ***Systems engineering***, and more recently model-based systems engineering, emerged as a practical and interdisciplinary engineering discipline that enables the successful realization of complex systems from concept, through design, to full implementation. It is well-equipped to deal with systems of ever-greater complexity; be they for the greater interaction within these systems or because of the expanding heterogeneity they demonstrate in their structure and function. Notable human achievements like sending a man to the moon or landing on the surface of Mars can certainly be attributed to the effective practice of systems engineering. Despite these achievements, model-based systems engineering, however, relies till today on graphical modeling languages that provide limited quantitative insight. In contrast, ***network science*** has emerged as a scientific discipline for quantitatively analyzing networks that appear in fields across the natural, social, and engineering sciences. And yet, network science, due to its reliance on graphs as a data structure, was often unable to address the explicit heterogeneity often encountered in the systems engineering field. Even the network science developments into ***multi-layer networks*** have been recognized to have significant limitations in modeling networked systems of arbitrary topology. Despite these methodological differences, these two informatic sciences have often tackled similar intellectual challenges. For example, both fields have contributed immensely to the knowledge of ***system life cycle properties*** like centrality, modularity, flexibility, sustainability, and resilience.

These two informatic sciences now face an even greater challenge. Not only are individual products being connected to form infrastructure networks, but these networks are forming interactions between each other to form systems-of-systems. The “smart grid", the energy-water nexus, the electrification of transport, are all good examples where one network system has fused

with another to form a new and much more capable system. This trend is only set to continue. Taken to its finality, it leads us to the pressing need to understand and implement smart cities as a platform upon which to integrate all of these efforts. Naturally, the methodological and theoretical limitations of model-based systems engineering and network science must be overcome to gain truly novel insight into the development of smart cities.

It is in this context, that we have written this book. *A Hetero-functional Graph Theory for Modeling Interdependent Smart City Infrastructure* lays a theoretical foundation that intellectually resembles a fusion of model-based systems engineering and network science. Hetero-functional graph theory relies on multiple graphs as data structures so as to support quantitative analysis. It also explicitly embodies the heterogeneity of conceptual and ontological constructs found in model-based systems engineering. Its application to interdependent smart city infrastructures of arbitrary topology presents a highly demanding use case.

Where did hetero-functional graph theory come from?

While hetero-functional graph theory can be viewed as an intellectual fusion of model-based systems engineering and network science, its origins are found elsewhere. Although not called as such at the time, the theory originated from the automated mass-customized production system literature. Production systems, as their own class of engineering system, present some unique modeling challenges. They can have a nearly arbitrary size, an unlimited diversity of production capabilities, and an almost infinite number of product variants. They also demonstrate a consistently changing structure and behavior. The need to compete in dynamic marketplaces with product variants of increasingly short product life-cycle drove mass-customized production systems to explicitly foster ***reconfigurability*** as a life-cycle property of their integrated automation solutions. To that end, Prof. Farid's doctoral dissertation *reconfigurability measurement in automated manufacturing systems* (2007) specifically developed a quantitative measure of reconfigurability. Perhaps unsurprisingly to a network scientist, it used a ***design structure matrix*** as a type of graph to address the ease of reconfiguration. Furthermore, and unsurprisingly to a systems engineer, it addressed the allocation of function to form as the ***central question of engineering design***. It drew the concept of a ***knowledge base*** from the ***Axiomatic Design*** literature and quantified it as a bipartite graph. Beyond these considerations, however, the reconfigurability measurement of mass-customized production systems needed to specifically address ***heterogeneity*** as its essential characteristic. Finally, because automation was an essential aspect of mass-customized production systems, the research was explicitly ***cyber-physical***.

In 2010, the Laboratory for Intelligent Integrated Networks of Engineering Systems (LIINES) was founded with a research program devoted to the sustainability and resilience of intelligent energy systems. Several research themes were launched year after year; first smart power grids, then energy-water nexus, then electrified transportation systems, and then industrial energy management. Each of these represented engineering systems where two intelligent energy systems were integrated. As work in each research theme developed, a pattern emerged. While each new application had its peculiarities that required enhancements to hetero-functional graph theory, the graph structures originally developed in the Farid dissertation could be used generically across multiple application domains. It is around this time that the LIINES research really began to ***converge*** and several publications sought to specifically state the cross-domain applicability of hetero-functional graph theory. The article entitled: *Static Resilience of Large Flexible Engineering Systems: Axiomatic Design Model and Measures* (2015) specifically demonstrated cross-domain applicability, addressed resilience as a life-cycle property, and acknowledged its roots in Axiomatic Design. In the following year, the book *Axiomatic Design in Large Systems* (2016) used the term ***Hetero-functional network*** for the first time. The first chapter explicitly links the Axiomatic Design literature to hetero-functional graph theory and engineering systems. Since that time, applications of hetero-functional graph theory have continued to expand. Prof. Khayal's research has applied hetero-functional graph theory to personalized healthcare delivery systems and W.C.H. Schoonenberg has sought to integrate these efforts into interdependent smart city infrastructures.

The Goal of this Book

Consequently, the goal of this book is to present, in one volume, a consistent hetero-functional graph theoretic treatment of interdependent smart city infrastructures as an ***overarching*** application domain of engineering systems. Naturally, in doing so, the work seeks to reconcile over a decade of research; including the many enhancements that came from tackling new and exciting application domains. Over the course of the text, we have made every effort to provide historical footnotes of how the theory has developed over that time. We have many hopes for the broad appeal of this work. To the systems engineering community, we hope that hetero-functional graph theory will be accepted as a quantification of many of the structural concepts found in model-based systems engineering languages like SysML. To the network science community, we hope to present a new view as how to construct graphs with fundamentally different meaning and insight. Finally, it is our hope that hetero-functional graph theory serves to overcome many of the theoretical

and modeling limitations that have hindered our ability to systematically understand the structure and function of smart cities.

What is in this Book?

This book is organized into seven chapters:

- Chapter 1 introduces the work in terms of the practical need to address smart cities as a pressing grand challenge. It also identifies the original contributions of the book and outlines how its argument evolves in the following chapters.
- Chapter 2 then turns to present the theoretical need for hetero-functional graph theory. An extensive discussion of multi-layer networks is provided. Its limitations are identified by means of a simple example of a hypothetical four layer network.
- Chapter 3 orients the reader with hetero-functional graph theory preliminaries. The fundamental ontological concepts of soundness, completeness, lucidity, and laconicity are presented as means by which to formally assess the fidelity of a model. Multi-layer networks are found to lack completeness and lucidity. The remainder of the chapter relates the systems engineering foundations for hetero-functional graph theory. In particular, it focuses on the concept of system architecture at the instantiated, reference, and meta levels of abstraction.
- Chapter 4 relates hetero-functional graph theory rigorously as an intellectual fusion of model-based systems engineering and graph theory in terms of its seven constitutent mathematical models. Simple examples are provided for each of these so as to demonstrate the conceptual links with SysML as a model-based systems engineering language. Formal definitions of all concepts are also provided so as to facilitate an explicit discussion of the underlying ontological structure.
- Chapter 5 then applies hetero-functional graph theory to an interdependent smart city infrastructure test case called "*Trimetrica*". One feature of "*Trimetrica*" is its significant heterogeneity of function. The chapter demonstrates the construction a single system adjacency matrix for such a heterogeneous system. It subsequently discusses how this demonstration overcomes many limitations found in the multi-layer network literature.
- Chapter 6 serves to point the reader to further applications of hetero-functional graph theory. In particular, it summarizes its contributions to (1) mass-customized production systems (2) transportation systems (3) electric power systems (4) electrified transportation systems (5) microgrid-enabled production systems (6) personalized healthcare delivery systems. Along the way, the chapter highlights how hetero-functional graph theory can be used to create dynamical system simulation models and study life cycle properties.

- Chapter 7 brings the book to a conclusion. It discusses some fertile areas for future research including the quantitative understanding of life cycle properties, the treatment of cyber-physical systems, and the application of the network science literature on hetero-functional graphs.

In all, these seven chapters provide the reader with a rigorous introduction to hetero-functional graph theory so as to begin making independent contributions to the literature.

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Nomenclature

Ontological Symbols

$\mathcal{A}$	Abstraction, equals the mental conceptualization of the modeller. *p. 14*
$\mathcal{C}$	Domain Conceptualization, equals the understanding of reality. *p. 14*
$\mathcal{D}$	Real Domain, equals reality. *p. 14*
$\mathcal{L}$	Language, equals the description of reality. *p. 14*
$\mathcal{M}$	Model, equals the description of the abstraction. *p. 14*

Sets

$\overline{R}$	Set of Aggregated Resources *p. 37*
$\overline{R}_P$	Set of Aggregated Physical Resources *p. 52*
B	Set of Independent Buffers *p. 31*
B_S	Set of Buffers *p. 32*
H	Set of Transportation Resources *p. 31*
L	Set of Services *p. 59*
M	Set of Transformation Resources *p. 31*
M_{l_i}	Set of arcs: (service states to service activities) and (service activities to service states) *p. 61*
P	Set of System Processes *p. 32*
P_η	Set of Transportation Processes *p. 32*
P_μ	Set of Transformation Processes *p. 32*
P_Q	Set of Decision Algorithms *p. 50*
$P_{\bar{\eta}}$	Set of Refined Transportation Processes *p. 33*
P_γ	Set of Holding Processes *p. 33*
Q	Set of Cyber-Resources *p. 50*
Q_D	Set of Dependent Cyber-Resources *p. 50*
Q_I	Set of Independent Cyber-Resources *p. 50*
R	Set of System Resources *p. 31*
R_P	Set of Physical Resources *p. 50*
S_{l_i}	Set of Places describing the service states for product l_i *p. 61*
W_{l_i}	Set of Weights on the arcs describing the probabilities *p. 61*
$\mathcal{E}$	Set of System Actions *p. 37*

$\mathcal{E}_S$ Set of Structural Degrees of Freedom ..... p. 40
$\mathcal{E}_{l_i}$ Set of Service Activities in Service l_i ..... p. 59
$\mathcal{E}_{LH}$ Set of Service Transportation Degrees of Freedom ..... p. 69
$\mathcal{E}_{LM}$ Set of Service Transformation Degrees of Freedom ..... p. 69
$\mathcal{E}_{LS}$ Set of Service Degrees of Freedom ..... p. 69
$\mathcal{Z}$ Set of System Activity Strings ..... p. 45

Set Elements

b Independent buffer in set of independent buffers B ..... p. 31
b_{sy_1} Origin buffer y_1 in set of buffers B_S ..... p. 32
b_{sy_2} Destination buffer y_2 in set of buffers B_S ..... p. 32
e_{wv} System action w, v in set of system actions $\mathcal{E}$ ..... p. 37
e_{xl_i} Service activity x in Service l_i ..... p. 59
h Transporter in set of transporters H ..... p. 31
l_i Service i in the set of services L ..... p. 59
m Machine in set of machines M ..... p. 31
p_Q Decision algorithm in the set of decision algorithms P_Q ..... p. 50
$p_{\bar{\eta}\varphi}$ Refined transportation process φ in set of refined transportation processes $P_{\bar{\eta}}$ ..... p. 34
$p_{\eta u}$ Transportation process u in set of transportation processes P_η . p. 32
$p_{\gamma g}$ Holding process g in set of holding processes P_γ ..... p. 33
$p_{\mu j}$ Transformation process j in set of transformation processes P_μ p. 32
q Cyber-resource in the set of cyber-resources Q ..... p. 50
r Resource in set of system resources R ..... p. 31
z_{χ_1,χ_2} String of two sequential activities χ_1 and χ_2 in set of strings $\mathcal{Z}$ p. 45

Indices

χ Index in $[1 \dots \sigma(R)\sigma(P)]$ ..... p. 45
ψ Index of the elements in the set of structural degrees of freedom $\mathcal{E}_S$ p. 47
φ Index of refined transportation process $p_{\bar{\eta}\varphi}$ in $P_{\bar{\eta}}$ ..... p. 34
g Index of holding process $p_{\gamma g}$ in set of holding processes P_γ ... p. 33
i Index of Service l_i in the set of services L ..... p. 59
j Index of transformation process $p_{\mu j}$ in set of transformation processes P_μ ..... p. 32
k Index of time ..... p. 61
u Index of transportation process $p_{\eta u}$ in set of transportation processes P_η ..... p. 32
v Index of physical resource p_w in set of physical resources R ... p. 37
w Index of system process p_w in set of system processes P ..... p. 37
x Index of service activity e_{xl_i} in the set of service activities $\mathcal{E}_{l_i}$ .. p. 59
y_1 Index of origin buffer b_{sy_1} in set of buffers B_S ..... p. 32
y_2 Index of destination buffer b_{sy_2} in set of buffers B_S ..... p. 32

Mathematical Symbols

$\Lambda_{\gamma i}$ Service Transportation Feasibility Matrix for product l_i ..... p. 66
$\Lambda_{\mu i}$ Service Transformation Feasibility Matrix for product l_i ..... p. 66
Λ_{Hi} Transportation service selector matrix ..... p. 69

Λ_{HL} Transportation service line selector matrix . . . . . p. 69
Λ_{Hxi} Transportation service activity selector matrix . . . . . p. 69
Λ_i Service Feasibility Matrix for product l_i . . . . . p. 67
Λ_{Mi} Transformation service selector matrix . . . . . p. 69
Λ_{ML} Transformation service line selector matrix . . . . . p. 69
Λ_{Mxi} Transformation service activity selector matrix . . . . . p. 69
Λ_{SHi} System Transportation Service selector matrix . . . . . p. 69
Λ_{SHxi} System Transportation service activity selector matrix . . . . . p. 69
Λ_{Si} System Transformation Service selector matrix . . . . . p. 69
Λ_{SL} System Service Line selector matrix . . . . . p. 69
Λ_{SMxi} System Transformation service activity selector matrix . . . . . p. 69
Λ_{Sxi} System Transformation and Transportation service activity selector matrix . . . . . p. 69
$\mathbb{1}^n$ Ones-vector of length n . . . . . p. 38
$\mathbb{A}$ System Adjacency Matrix . . . . . p. 72
$\mathbb{A}_L$ System Service Adjacency Matrix . . . . . p. 72
$\mathbb{A}_{\rho C}$ System Controller Agency Matrix . . . . . p. 72
$\mathbb{A}_{\rho L}$ Service System Feasibility Matrix . . . . . p. 72
$\mathbb{A}_{C\rho}$ Controller System Agency Matrix . . . . . p. 72
$\mathbb{A}_{L\rho}$ System Service Feasibility Matrix . . . . . p. 72
$\mathbb{P}_S$ A (non-unique) projection matrix for the vectorized knowledge base p. 47
$\overline{A}_Q$ Independent Controller Agency Matrix that shows jurisdiction of Q_I over R_P . . . . . p. 52
Φ_T State Transition Function for a timed Petri net . . . . . p. 61
$\widetilde{A}_\rho$ Hetero-functional Adjacency Matrix after elimination of row and column sparsity . . . . . p. 47
Ξ Resource Aggregation Matrix . . . . . p. 37
A_C Controller Adjacency Matrix . . . . . p. 57
A_Q Controller Agency Matrix . . . . . p. 52
A_S System Concept . . . . . p. 40
A_ρ Hetero-functional Adjacency Matrix . . . . . p. 44
DOF_ρ Sequence-Dependent Degrees of Freedom . . . . . p. 47
DOF_H Transportation Degrees of Freedom . . . . . p. 41
DOF_M Transformation Degrees of Freedom . . . . . p. 41
DOF_S Structural Degrees of Freedom . . . . . p. 40
$DOF_{HH\rho}$ Measure of Type IV Sequence-Dependent Production Degrees of Freedom . . . . . p. 45
$DOF_{HM\rho}$ Measure of Type III Sequence-Dependent Production Degrees of Freedom . . . . . p. 45
DOF_{LH} The number of transportation capabilities utilized by all services p. 70
DOF_{LM} The number of transformation capabilities utilized by all services p. 70
DOF_{LS} The number of capabilities utilized by all services . . . . . p. 70

$DOF_{MH\rho}$ Measure of Type II Sequence-Dependent Production Degrees of Freedom ..... p. 45
$DOF_{MM\rho}$ Measure of Type I Sequence-Dependent Production Degrees of Freedom ..... p. 45
I^n Identity Matrix of size $n \times n$ ..... p. 52
J_γ Holding Knowledge Base ..... p. 38
J_H Transportation Knowledge Base ..... p. 38
J_M Transformation Knowledge Base ..... p. 38
J_S System Knowledge Base ..... p. 37
$J_{\bar{H}}$ Refined Transportation Knowledge Base ..... p. 38
J_ρ System Sequence Knowledge Base ..... p. 44
$J_{HH\rho}$ Type IV Sequence-Dependent Knowledge Base ..... p. 45
$J_{HM\rho}$ Type III Sequence-Dependent Knowledge Base ..... p. 45
$J_{MH\rho}$ Type II Sequence-Dependent Knowledge Base ..... p. 45
$J_{MM\rho}$ Type I Sequence-Dependent Knowledge Base ..... p. 45
K_ρ System Sequence Constraints Matrix ..... p. 44
K_M Transformation Constraints Matrix ..... p. 40
K_S System Constraints Matrix ..... p. 40
$K_{\bar{H}}$ Refined Transportation Constraints Matrix ..... p. 40
$K_{HH\rho}$ Type IV Sequence-Dependent Constraints Matrix ..... p. 45
$K_{HM\rho}$ Type III Sequence-Dependent Constraints Matrix ..... p. 45
$K_{MH\rho}$ Type II Sequence-Dependent Constraints Matrix ..... p. 45
$K_{MM\rho}$ Type I Sequence-Dependent Constraints Matrix ..... p. 45
N_{l_i} Service Net for product l_i ..... p. 61
$Q_{El_i}[k]$ Marking of Service Transitions for product l_i at time k ..... p. 61
Q_{l_i} Petri net marking representing the set of service states ..... p. 61
$Q_{Sl_i}[k]$ Marking of Service States for product l_i at time k ..... p. 61
$U^+_{l_i}[k]$ Binary Input Firing Vector for product l_i at time k ..... p. 61
$U^-_{l_i}[k]$ Binary Output Firing Vector for product l_i at time k ..... p. 61
z_{l_i} Sequence of Service Activities $e_{xl_i} \forall x \in [1,\ldots,\sigma(\mathcal{E}_{l_i})]$ ..... p. 60
$\mathcal{F}_v$ Resource Flexibility ..... p. 38
$\mathcal{R}_w$ Process Redundancy ..... p. 37

Mathematical Operators

$()^V$ Shorthand for vectorization (i.e. vec()) ..... p. 45
$\circledast$ Matrix Aggregation Operator ..... p. 37
$\langle A, B \rangle_F$ Frobenius Product of matrices A and B ..... p. 40
$\odot$ Matrix boolean multiplication ..... p. 37
$\otimes$ Kronecker Product ..... p. 38
$\sigma()$ The size of the set (). ..... p. 32
$\bigtimes$ Cartesian Product ..... p. 33