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DRAFT: AN ENGINEERING SYSTEMS SENSITIVITY ANALYSIS MODEL FOR HOLISTIC ENERGY-WATER NEXUS PLANNING

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ABSTRACT

The energy-water nexus is an area of increasing global concern and research. In several existing publications on the subject, the challenges of water use for power plant cooling and energy use for water supply are handled seperately. There is however also a need to consider the totality of interactions between the different elements of the engineered water and electricity systems, thus creating a system-of-systems model. A model of this form integrates water use for electricity supply and electricity use for water supply into a single framework, thus elucidating a wide range of interactions which can be influenced by policy and management decisions to achieve desired objectives. An engineering model capturing these interactions and based on first-pass models of the underlying physics of the various coupling and boundary points has been developed in previous work. In this work,the Jacobian of the resulting system of equations has been determined for a particular illustrative case. This Jacobian enables a sensitivity analysis of the inputs and outputs of this system-of-systems to changes in water and electricity demand to be carried out. As a concrete example, the Jacobian is used to examine the effect of a 10 % growth in both electricity and water demand on the set of system inputs and outputs.

1 INTRODUCTION

The energy-water nexus is under increasing strain due to population growth, economic growth and climate change [1]. Resultantly, a number of discussions on the energy-water nexus have been published in recent years. Overviews of the various challenges related to the nexus, as well as discussions of various policy options for the amelioration of the risks can be found in [1–7]. Empirical evaluations of the electricity-intensity of water technologies and the water-intensity of electricity technologies have been reported and analysed in [8–13]. Efforts have been made towards physics-based models in [14, 15] in which formulations for estimating water use by thermal power plants based on the heat balance of the plant have been derived. An integrated operational view of the water and power networks has also been presented as a simultaneous co-optimization for the economic dispatch of power and water [16–18].

Increasingly there has also been a focus on the development of tools for integrated planning of electricity and water supply systems. A decision support system for the United States based on an underlying system dynamics model is described in [19]. The model enables the exploration of various water and electricity policies and relies on statistical relationships between the independent variables of population and economic growth and the dependent variables of electricity and water demand. Recent work [20] has interfaced the well known Regional Energy Deployment System (ReEDS) and Water Evaluation and Planning (WEAP) tools to create a platform for determining the water resource implications of different electricity sector development pathways. The platform uses empirical consumption and withdrawal coefficients reported in [11] for the interface.

This work attempts to contribute to the growing body of

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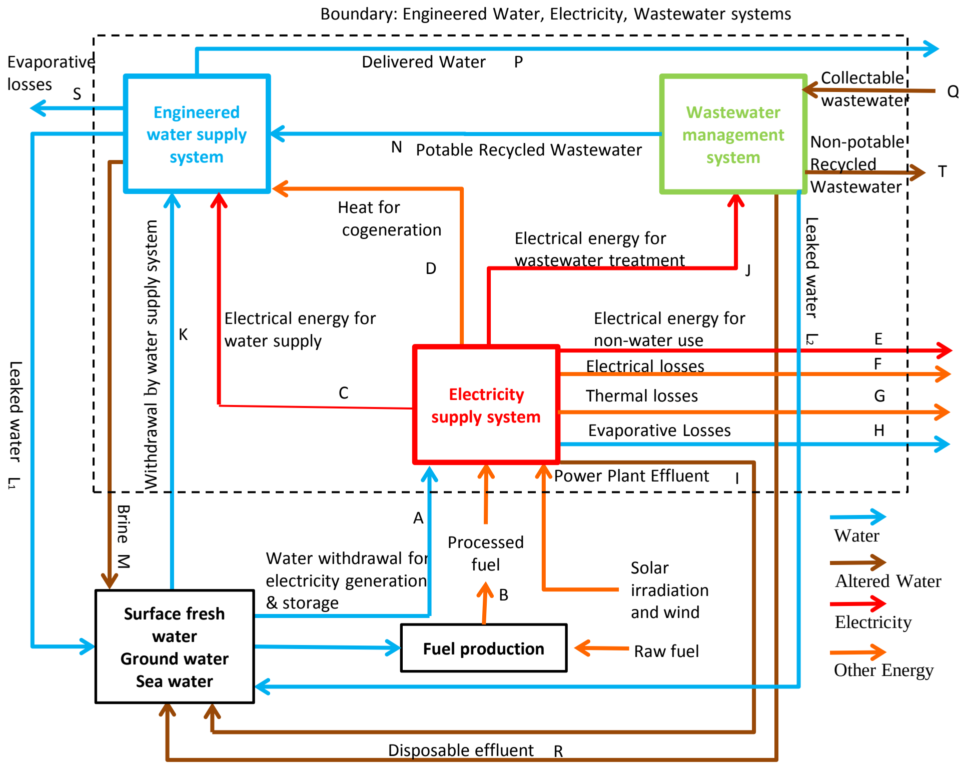

FIGURE 1. SYSTEM OF SYSTEMS BOUNDARY

knowledge around such integrated planning tools. In relation to a defined system-of-systems boundary encompassing the electricity, water and wastewater grids, the energy-water nexus can be viewed as an exchange of power in various energy domains between the interconnected electricity, water and wastewater grids as well as between each grid individually, and its environment. An engineering model capturing these power exchanges and based on first-pass models of the underlying physics of the various coupling and boundary points has been developed in previous work [21, 22]. The model equations are summarized in Eqns 16, 17 and 18 in Appendix A. In this work the Jacobian of this system of equations is derived and applied as a sensitivity analysis tool.

The paper proceeds as follows. First, a system-of-systems boundary is presented in Section 2 and the modeling objective clarified. In Sections 3, 4 and 5 Jacobians of input-output models of the electricity, water and wastewater systems resepctively are developed. These are then combined into the desired system-of-systems Jacobian in Section 6 which is applied to a hypothetical illustrative case in Section 7.

2 SYSTEM BOUNDARY AND CONTEXT

Figure 1 chooses the system boundary around the three engineering systems of electricity, water and wastewater [21, 23, 24]. It also depicts the high level flows of matter and energy between them and the natural environment. Electricity, potable water, and wastewater are all primarily stationary within a region's infrastructure. In contrast, the traditional fuels of natural gas, oil, and coal are open to trade. Consequently, the fuel processing function, though it has a significant water footprint, is left outside of the system boundary. An advantage of this choice of system boundary is that the three engineering systems all fall under the purview of grid operators, and in some nations, such as the United Arab Emirates, all three grid operations are united within a single semi-private organization.

Figure 1 makes possible the identification of a set of independent, $\mathbf{X}$, and dependent, $\mathbf{Y}$, power variables at the delineated system boundary, which are listed in Tab 1 alongside their respective identifiers in the figure. The reader is referred to the nomenclature section for a definition of each of the listed variables. A combined engineering systems model of the form $\mathbf{f}(\mathbf{X},\mathbf{Y}) = \mathbf{0}$ can then be developed from which a Jacobian $\mathbf{dY}/\mathbf{dX}$ can be determined and evaluated at a point $(\mathbf{X_0},\mathbf{Y_0})$. It is desired to use this Jacobian as a sensitivity analysis tool, to determine the effects of small changes in $\mathbf{X}$ on $\mathbf{Y}$:

$$\Delta\mathbf{Y} \approx \left.\frac{\mathbf{dY}}{\mathbf{dX}}\right|_{\mathbf{X_0}} \Delta\mathbf{X} \tag{1}$$

TABLE 1. INDEPENDENT AND DEPENDENT VARIABLES AT SYSTEM OF SYSTEMS BOUNDARY

Identifier in Fig 1	X	Y
A	$\mathbf{\Delta T_W}$	$\mathbf{Q_G^{in}}$
B	$\mathcal{H}_\mathbf{G}$	$\mathbf{\dot{M}_G}$
E	$\mathbf{I_{L0}}$	$\mathbf{V_L}$
H	$\mathbf{Q_G^{evap}}$	-
I	-	$\mathbf{Q_G^{out}}$
K	-	$\mathbf{Q_F^{*}}$
L	p_{atm}	$\mathbf{Q_{Jl}}, \mathbf{Q_{El}}$
M	-	$\mathbf{Q_F^{brine}}$
P=Q	$\mathbf{Q_J}$	$\mathbf{P_J}$
R	-	$\mathbf{Q_D^{disp}}$
S	$\mathbf{Q_S^{evap}}$	-
T	$\mathbf{Q_E}$	$\mathbf{P_E}$

Due to the complexity associated with developing a single system of equations, $\mathbf{f}(\mathbf{X},\mathbf{Y}) = \mathbf{0}$, describing all three systems, a set of input-output models for each system has been developed individually [21, 22]. These will be differentiated individually in the following sections, and the different individual system Jacobians subsequently combined.

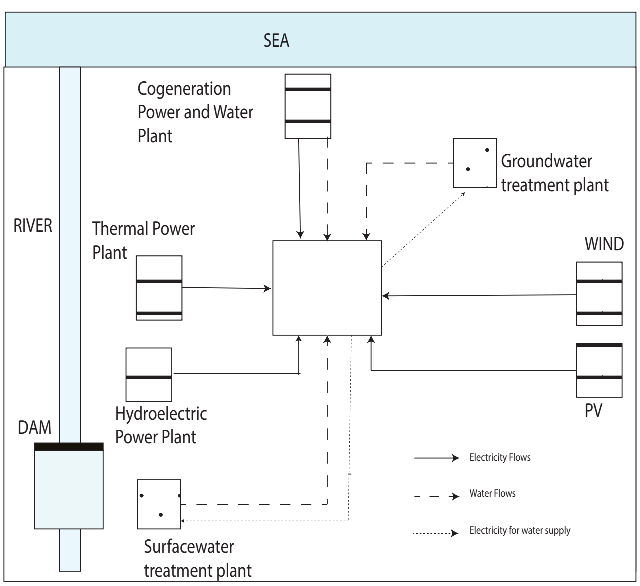

FIGURE 2. ILLUSTRATIVE CASE

The sensitivity analysis will be demonstrated through a simple hypothetical case conceptually illustrated in Fig 2. In summary it represents a region served by three different water supply options and five different electricity supply options. The electricity network is modelled by means of the IEEE 14 bus test case, with slight modifications made to incorporate more generators, where as the water supply plants are simply modelled as being equidistant from an aggregated demand node. No wastewater management system is modelled and thus it is assumed that all wastewater output from the demand node is disposed of in an untreated form. In previous work [21, 22], this test case has been used to demonstrate the solution of a combined engineering systems model of the electricity, water and wastewater networks. The solution presented provides the base operating point $(\mathbf{X_0}, \mathbf{Y_0})$ for the sensitivity analysis in this work. Full details of the system parameters for this case can be found in this previous work, however those that are germane to this discussion are reported as necessary and their associated symbols are defined in the nomenclature. The values of the elements of $\mathbf{X_0}$ and $\mathbf{Y_0}$ required for evaluation of the Jacobian are also reported as necessary.

A wide range of variables across the three systems of interest are modelled in this work. The reader is referred to the nomenclature section for a full description of variables used in the models in Sections 3, 4 and 5. As a general guide however the letters used to represent the different variables are as follows: (1) **I** - Current, (2) **V** - Voltage, (3) **Q** - Volumetric Flow Rate, (4) **P** - Pressure, (5) **H** - Specific Enthalpy, (6) **T** - Temperature, and (7) **M** - Fuel consumption rate. Additionally, the subscripts of these variables are denoted by the set of facilities to which they belong: (1) **F** - Nodes in the water distribution network with known pressure (water treatment plants), (2) **J** - Nodes in water distribution network with unknown pressure, (3) **E** - Nodes in non-potable recycled wastewater distribution network with unknown pressure, (4) $\mathcal{P}$ - Water distribution links (includes both pipes and distribution pumps), (5) **N** - Non-potable recycled wastewater distribution links (includes both pipes and distribution pumps), (6) **G** - Generators, (7) **L** - Electrical loads, (8) $\mathcal{T}$ - Electrical transmission lines, (9) **W** - Water sources i.e. rivers, lakes, aquifers or the sea, (10) **S** - Water storage units, both tanks and artificial reserovirs, and (11) **D** - Wastewater treatment plants.

It must be clarified that electricity supply system and electrical energy drawn by the water and wastewater systems, have been modelled in terms of currents and voltages in this work. This is a result of a decision to apply the bond graph methodology [25, 26] to develop the equations in Appendix A in previous work [21, 22]. The bond graph methodology presents a number of practical advantages. First, it readily facilitates the inter-energy-domain modeling necessitated by the heterogeneous nature of the energy-water nexus. Next, it clearly distinguishes between directionality and causality and as a result, clarifies the dependence and independence of variables. Finally the bond graph methodology facilitates the development of more complex models to incorporate previously neglected functionality or to introduce dynamic effects. Nevertheless the approach presented in this work can readily be adapted to more conventional models of the electricity supply system.

3 ELECTRICITY SUPPLY SYSTEM

The independent and dependent variables at the boundary of the electricity supply system (System $\mathcal{E}$) are identified as the following:

$$\begin{aligned} \mathbf{X}_{\mathcal{E}} &= \{\mathbf{I_{L0}}, \mathbf{I_{LW}}, \Delta\mathbf{T_W}, \mathcal{H}_\mathbf{G}, \mathbf{P_W}, \lambda_\mathbf{G}, \Gamma_\mathbf{G}, \mathbf{Q_G}^{evap}\} \\ \mathbf{Y}_{\mathcal{E}} &= \{\mathbf{V_L}, \mathbf{Q_G^{in}}, \mathbf{Q_G^{out}}, \dot{\mathbf{M}}_\mathbf{G}\} \end{aligned} \quad (2)$$

Referring to Eqn 16 the derivative of the dependent variables with respect to the independent variables for the electricity system can be determined as in Eqn 3. $\mathcal{F}_{\mathbf{G_1}}$ and $\mathcal{F}_{\mathbf{G_2}}$ are fairly complex functions which include polynomial regressions of portions of the steam tables. The partial derivatives in Eqn 3 can be determined at the point $(\mathbf{X_0}, \mathbf{Y_0})$ defined by the illustrative case and thus $\mathbf{dY}_{\mathcal{E}}/\mathbf{dX}_{\mathcal{E}}$ at this point determined. The resulting matrix is not shown due to space constraints but is applied in Sections 6 and 7.

4 ENGINEERED WATER SUPPLY SYSTEM

The independent and dependent variables at the boundary of the engineered water supply system (System $\mathcal{W}$) are identified as the following:

$$\begin{aligned} \mathbf{X}_{\mathcal{W}} &= \{\mathbf{Q_J}, \mathbf{Q_S^{evap}}, \Pi_\mathbf{W}, \mathbf{P_W}, \mathbf{V_F}\} \\ \mathbf{Y}_{\mathcal{W}} &= \{\mathbf{P_J}, \mathbf{I}_{\mathcal{P}}, \mathbf{Q_{Jl}}, \mathbf{Q}_{\mathcal{P}}, \mathbf{Q_F}, \mathbf{I_F}, \mathbf{P_F}, \Delta\mathbf{H_F}, \mathbf{Q_F^{brine}}, \mathbf{Q_F^*}\} \end{aligned} \quad (4)$$

The previously developed system model (Eqn 17) is an implicit equation of the form $\mathbf{f}_{\mathcal{W}}(\mathbf{X}_{\mathcal{W}}, \mathbf{Y}_{\mathcal{W}}) = \mathbf{0}$ from which it can be determined that $d\mathbf{Y}_{\mathcal{W}}/d\mathbf{X}_{\mathcal{W}}$ is given by:

$$\frac{d\mathbf{Y}_{\mathcal{W}}}{d\mathbf{X}_{\mathcal{W}}} = \left[\frac{\partial \mathbf{f}_{\mathcal{W}}}{\partial \mathbf{Y}_{\mathcal{W}}}\right]^{-1} \frac{\partial \mathbf{f}_{\mathcal{W}}}{\partial \mathbf{X}_{\mathcal{W}}} \quad (5)$$

The partial derivatives $\partial\mathbf{f}_{\mathcal{W}}/\partial\mathbf{Y}_{\mathcal{W}}$ and $\partial\mathbf{f}_{\mathcal{W}}/\partial\mathbf{X}_{\mathcal{W}}$ can be determined from Eqn 17 as the following:

$$\frac{\partial \mathbf{f}_{\mathcal{W}}}{\partial \mathbf{Y}_{\mathcal{W}}} = \begin{bmatrix} -\beta_J{}' & 0 & \mathbb{1}_\mathbf{J} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbb{1}_\mathbf{J} & -\mathbf{B_J}{}^\dagger & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{B_J} & 0 & 0 & -\mathbf{R}_{\mathcal{P}}{}' & 0 & 0 & \mathbf{B_F} & 0 & 0 & 0 \\ 0 & \mathbb{1}_{\mathcal{P}} & 0 & -\mathbf{K}_{\mathcal{P}}\Psi_{\mathcal{P}}{}' & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{B_F}{}^\dagger & -\mathbb{1}_\mathbf{F} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{K_6} & -\mathbb{1}_\mathbf{F} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{K_9} & 0 & -\mathbb{1}_\mathbf{F} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{Z_F} & 0 & 0 & -\mathbb{1}_\mathbf{F} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{RR_F} & 0 & 0 & 0 & -\mathbb{1}_\mathbf{F} & 0 \\ 0 & 0 & 0 & 0 & \mathbb{1}_\mathbf{F} & 0 & 0 & 0 & \mathbb{1}_\mathbf{F} & -\mathbb{1}_\mathbf{F} \end{bmatrix} \quad (6)$$

$$\frac{\partial \mathbf{f}_{\mathcal{W}}}{\partial \mathbf{X}_{\mathcal{W}}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \mathbb{1}_\mathbf{J} & \mathbf{C_{JS}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{K_8}\mathbf{C_{WF}}{}^\dagger & 0 & \mathbf{K_7} \\ 0 & 0 & 0 & \mathbf{C_{WF}}{}^\dagger & \mathbf{K_{10}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

The values of the various elements of $\partial\mathbf{f}_{\mathcal{W}}/\mathbf{Y}_{\mathcal{W}}$ and $\partial\mathbf{f}_{\mathcal{W}}/\mathbf{X}_{\mathcal{W}}$ for the particular case described in Section 2 are given in Tab 2. These values can be substituted into Eqns 6 and 7 and used to determine $d\mathbf{Y}_{\mathcal{W}}/d\mathbf{X}_{\mathcal{W}}$ for the particular case by means of Eqn 5. As in the previous section, the resulting matrix is not shown due to space constraints but is applied in Sections 6 and 7.

5 WASTEWATER MANAGEMENT SYSTEM

The independent and dependent variables at the boundary of the wastewater management system (System $\mathcal{WW}$) are identified as the following:

$$\begin{aligned} \mathbf{X}_{\mathcal{WW}} &= \{\mathbf{Q_D}, \mathbf{Q_D^{rec1}}, \mathbf{Q_E}, \mathbf{P_D}\} \\ \mathbf{Y}_{\mathcal{WW}} &= \{\mathbf{P_E}, \mathbf{I_N}, \mathbf{Q_{El}}, \mathbf{Q_N}, \mathbf{Q_D^{rec2}}, \mathbf{I_D}, \mathbf{Q_D^{disp}}\} \end{aligned} \quad (8)$$

The implicit system of equations describing the input-output model for the wastewater system (Eqn 18) can be treated in the same manner as the engineered water supply system with $\partial\mathbf{f}_{\mathcal{WW}}/\partial\mathbf{Y}_{\mathcal{WW}}$ and $\partial\mathbf{f}_{\mathcal{WW}}/\partial\mathbf{X}_{\mathcal{WW}}$ determined as the following:

$$\frac{\mathbf{dY}_{\mathcal{E}}}{\mathbf{dX}_{\mathcal{E}}} = \begin{bmatrix} -\mathbf{A_1} & -\mathbf{A_1} & 0 & 0 & \mathbf{A_2}\frac{\partial \mathbf{V_G}}{\partial \mathbf{P_W}} & \mathbf{A_2}\frac{\partial \mathbf{V_G}}{\partial \lambda_\mathbf{G}} & \mathbf{A_2}\frac{\partial \mathbf{V_G}}{\partial \Gamma_\mathbf{G}} & 0 \\ \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \mathbf{I_{L0}}} & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \mathbf{I_{LW}}} & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \Delta \mathbf{T_W}} & 0 & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \mathbf{P_W}} & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \lambda_\mathbf{G}} & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \Gamma_\mathbf{G}} & 0 \\ \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \mathbf{I_{L0}}} & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \mathbf{I_{LW}}} & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \Delta \mathbf{T_W}} & 0 & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \mathbf{P_W}} & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \lambda_\mathbf{G}} & \frac{\partial \mathcal{F}_{\mathbf{G_1}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \Gamma_\mathbf{G}} & -\mathbb{1}_\mathbf{G} \\ \frac{\partial \mathcal{F}_{\mathbf{G_2}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \mathbf{I_{L0}}} & \frac{\partial \mathcal{F}_{\mathbf{G_2}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \mathbf{I_{LW}}} & 0 & \frac{\partial \mathcal{F}_{\mathbf{G_2}}}{\partial \mathcal{H}_\mathbf{G}} & \frac{\partial \mathcal{F}_{\mathbf{G_2}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \mathbf{P_W}} & \frac{\partial \mathcal{F}_{\mathbf{G_2}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \lambda_\mathbf{G}} & \frac{\partial \mathcal{F}_{\mathbf{G_2}}}{\partial \mathbf{I_G}}\frac{\partial \mathbf{I_G}}{\partial \Gamma_\mathbf{G}} & 0 \end{bmatrix} \quad (3)$$

TABLE 2. ELEMENTS OF $\partial \mathbf{f}_{\mathcal{W}}/\partial \mathbf{Y}_{\mathcal{W}}$ & $\partial \mathbf{f}_{\mathcal{W}}/\partial \mathbf{X}_{\mathcal{W}}$ MATRICES FOR ILLUSTRATIVE CASE

Variable	Value
$\mathbb{1}_\mathbf{F}$	$diag(-1,-1,-1)$
$\mathbb{1}_\mathbf{J}$	$[1]$
$\mathbb{1}_\mathcal{P}$	$diag(1,1,1)$
$\beta'_\mathbf{J}$	$[-5.0637\times10^{-7}]$
$\mathbf{B_F} = \mathbf{B_F^\dagger}$	$diag(1,1,1)$
$\mathbf{B_J}$	$[-1;-1;-1]$
$\mathbf{B_J}^\dagger$	$[-1,-1,-1]$
$\mathbf{C_{WF}}$	$diag(1,1,1)$
$\mathbf{K_6}$	$diag(100,100,100)$
$\mathbf{K_7}$	$diag(0,0,0)$
$\mathbf{K_8}$	$diag(0,0,0)$
$\mathbf{K_9}$	$diag(0,0,0)$
$\mathbf{K_{10}}$	$diag(100,100,100)$
$\mathbf{K}_\mathcal{P}\Psi_P{}'(\mathbf{Q}_\mathcal{P})$	$diag(0,0,0)$
$\mathbf{R}'_\mathcal{P}(\mathbf{Q}_\mathcal{P})$	$diag(2.16\times10^3, 2.16\times10^3, 2.16\times10^3)$
$\mathbf{RR_F}$	$diag(0,0,1.86)$
$\mathbf{Z_F}$	$diag(0,0,720)$

$$\frac{\partial \mathbf{f}_{\mathcal{WW}}}{\partial \mathbf{Y}_{\mathcal{WW}}} = \begin{bmatrix} -\beta_E{}' & 0 & \mathbb{1}_\mathbf{E} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbb{1}_\mathbf{E} & -\mathbf{B_E}^\dagger & 0 & 0 & 0 \\ \mathbf{B_E} & 0 & 0 & -\mathbf{R_N}' & 0 & 0 & 0 \\ 0 & \mathbb{1}_\mathbf{N} & 0 & -\mathbf{K_N}\Psi_N{}' & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{B_D}^\dagger & -\mathbb{1}_\mathbf{D} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mathbb{1}_\mathbf{D} & 0 \\ 0 & 0 & 0 & 0 & -\mathbb{1}_\mathbf{D} & 0 & -\mathbb{1}_\mathbf{D} \end{bmatrix} \quad (9)$$

$$\frac{\partial \mathbf{f}_{\mathcal{WW}}}{\partial \mathbf{X}_{\mathcal{WW}}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbb{1}_\mathbf{E} & 0 \\ 0 & 0 & 0 & \mathbf{B_D} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mathbf{K_D} & 0 & 0 & 0 \\ \mathbb{1}_\mathbf{D} & -\mathbb{1}_\mathbf{D} & 0 & 0 \end{bmatrix} \quad (10)$$

Since a wastewater recycling system was not modelled in the illustrative case the first four rows of both the $\partial \mathbf{f}_{\mathcal{WW}}/\mathbf{Y}_{\mathcal{WW}}$ and $\partial \mathbf{f}_{\mathcal{WW}}/\mathbf{X}_{\mathcal{WW}}$ matrices are of zero dimension. Furthermore the fifth column of $\partial \mathbf{f}_{\mathcal{WW}}/\mathbf{Y}_{\mathcal{WW}}$ is also of zero dimension. The values of the various elements of $\partial \mathbf{f}_{\mathcal{WW}}/\mathbf{Y}_{\mathcal{WW}}$ and $\partial \mathbf{f}_{\mathcal{WW}}/\mathbf{X}_{\mathcal{WW}}$ that do exist for the particular case described in Section 2 are given in Tab 3. As in the previous sections, these values can be substituted into Eqns 9 and 10 and subsequently used to determine $d\mathbf{Y}_{\mathcal{WW}}/d\mathbf{X}_{\mathcal{WW}}$ for the particular case.

TABLE 3. ELEMENTS OF $\partial \mathbf{f}_{\mathcal{WW}}/\partial \mathbf{Y}_{\mathcal{WW}}$ and $\partial \mathbf{f}_{\mathcal{WW}}/\partial \mathbf{X}_{\mathcal{WW}}$ MATRICES FOR ILLUSTRATIVE CASE

Variable	Value
$\mathbb{1}_\mathbf{D}$	$[1]$
$\mathbf{K_D}$	$[0]$

6 COMBINED SYSTEM JACOBIAN

The set of independent variables at the system-of-systems boundary $\mathbf{X}$ is a subset of the union of the sets of independent variables at the boundaries of each of the individual systems. The same applies to the set of dependent variables $\mathbf{Y}$.

$$\begin{aligned} \mathbf{X} &\subset \{\mathbf{X}_{\mathcal{E}} \cup \mathbf{X}_{\mathcal{W}} \cup \mathbf{X}_{\mathcal{WW}}\} \\ \mathbf{Y} &\subset \{\mathbf{Y}_{\mathcal{E}} \cup \mathbf{Y}_{\mathcal{W}} \cup \mathbf{Y}_{\mathcal{WW}}\} \end{aligned} \tag{11}$$

Therefore if $\mathbf{Y}^* = \{\mathbf{Y}_{\mathcal{E}} \cup \mathbf{Y}_{\mathcal{W}} \cup \mathbf{Y}_{\mathcal{WW}}\}$ and $\mathbf{X}^* = \{\mathbf{X}_{\mathcal{E}} \cup \mathbf{X}_{\mathcal{W}} \cup \mathbf{X}_{\mathcal{WW}}\}$ are defined, the desired $\mathbf{dY}/\mathbf{dX}$ can be extracted from a matrix of the derivatives $\mathbf{dY}^*/\mathbf{dX}^*$ which is given by Eqn 12.

$$\frac{\mathbf{dY}^*}{\mathbf{dX}^*} = \begin{bmatrix} \frac{\mathbf{dY}_{\mathcal{E}}}{\mathbf{dX}_{\mathcal{E}}} & \frac{\partial \mathbf{Y}_{\mathcal{E}}}{\partial \mathbf{X}_{\mathcal{W}}} & \frac{\partial \mathbf{Y}_{\mathcal{E}}}{\partial \mathbf{X}_{\mathcal{WW}}} \\ \frac{\partial \mathbf{Y}_{\mathcal{W}}}{\partial \mathbf{X}_{\mathcal{E}}} & \frac{\mathbf{dY}_{\mathcal{W}}}{\mathbf{dX}_{\mathcal{W}}} & \frac{\partial \mathbf{Y}_{\mathcal{W}}}{\partial \mathbf{X}_{\mathcal{WW}}} \\ \frac{\partial \mathbf{Y}_{\mathcal{WW}}}{\partial \mathbf{X}_{\mathcal{E}}} & \frac{\partial \mathbf{Y}_{\mathcal{WW}}}{\partial \mathbf{X}_{\mathcal{W}}} & \frac{\mathbf{dY}_{\mathcal{WW}}}{\mathbf{dX}_{\mathcal{WW}}} \end{bmatrix} \tag{12}$$

The diagonal entries of this matrix have been determined in the previous three sections. In order to determine the unknown off-diagonal partial derivatives in Eqn 12, which represent coupling between the three systems, attention must be focussed on the equations characterizing the coupling shown in Fig 1, that is to say the equations that characterize energy and matter flows. C, J, N. Furthermore, recognizing that $Q = P$ in the same figure, the equations characterizing this interaction must also be differentiated. Table 4 provides the required equations and their associated non-zero derivatives. These derivatives are of independent variables of one system with respect to the dependent variables of another however they are readily used to determine the required off-diagonal entries in Eqn 12 if these entries are decomposed by means of the chain rule as shown in Eqn 13.

$$\begin{aligned} \frac{\partial \mathbf{Y}_{\mathcal{E}}}{\partial \mathbf{X}_{\mathcal{W}}} &= \frac{\mathbf{dY}_{\mathcal{E}}}{\mathbf{dX}_{\mathcal{E}}} \cdot \frac{\partial \mathbf{X}_{\mathcal{E}}}{\partial \mathbf{Y}_{\mathcal{W}}} \cdot \frac{\mathbf{dY}_{\mathcal{W}}}{\mathbf{dX}_{\mathcal{W}}} \\ \frac{\partial \mathbf{Y}_{\mathcal{E}}}{\partial \mathbf{X}_{\mathcal{WW}}} &= \frac{\mathbf{dY}_{\mathcal{E}}}{\mathbf{dX}_{\mathcal{E}}} \cdot \frac{\partial \mathbf{X}_{\mathcal{E}}}{\partial \mathbf{Y}_{\mathcal{WW}}} \cdot \frac{\mathbf{dY}_{\mathcal{WW}}}{\mathbf{dX}_{\mathcal{WW}}} \\ \frac{\partial \mathbf{Y}_{\mathcal{W}}}{\partial \mathbf{X}_{\mathcal{E}}} &= \frac{\mathbf{dY}_{\mathcal{W}}}{\mathbf{dX}_{\mathcal{W}}} \cdot \frac{\partial \mathbf{X}_{\mathcal{W}}}{\partial \mathbf{Y}_{\mathcal{E}}} \cdot \frac{\mathbf{dY}_{\mathcal{E}}}{\mathbf{dX}_{\mathcal{E}}} \\ \frac{\partial \mathbf{Y}_{\mathcal{W}}}{\partial \mathbf{X}_{\mathcal{WW}}} &= \frac{\mathbf{dY}_{\mathcal{W}}}{\mathbf{dX}_{\mathcal{W}}} \cdot \frac{\partial \mathbf{X}_{\mathcal{W}}}{\partial \mathbf{Y}_{\mathcal{WW}}} \cdot \frac{\mathbf{dY}_{\mathcal{WW}}}{\mathbf{dX}_{\mathcal{WW}}} \\ \frac{\partial \mathbf{Y}_{\mathcal{WW}}}{\partial \mathbf{X}_{\mathcal{E}}} &= \frac{\mathbf{dY}_{\mathcal{WW}}}{\mathbf{dX}_{\mathcal{WW}}} \cdot \frac{\partial \mathbf{X}_{\mathcal{WW}}}{\partial \mathbf{Y}_{\mathcal{E}}} \cdot \frac{\mathbf{dY}_{\mathcal{E}}}{\mathbf{dX}_{\mathcal{E}}} \\ \frac{\partial \mathbf{Y}_{\mathcal{WW}}}{\partial \mathbf{X}_{\mathcal{W}}} &= \frac{\mathbf{dY}_{\mathcal{WW}}}{\mathbf{dX}_{\mathcal{WW}}} \cdot \frac{\partial \mathbf{X}_{\mathcal{WW}}}{\partial \mathbf{Y}_{\mathcal{W}}} \cdot \frac{\mathbf{dY}_{\mathcal{W}}}{\mathbf{dX}_{\mathcal{W}}} \end{aligned} \tag{13}$$

The heat for cogeneration (D in Fig 1) has been modelled in previous work [21, 22] as being provided by steam from the exhaust of a condensing turbine. Such heat is in essence 'waste heat', and hence does not have an effect on the electricity generation process. In future refinements, the fact that this heating steam is typically taken from the exhaust of a back presssure turbine and therefore has an effect on the electricity generation process will be incorporated into the model and thus the equations characterizing D will also need to be differentiated.

The extraction of the desired $\mathbf{dY}/\mathbf{dX}$ matrix from the $\mathbf{dY}^*/\mathbf{dX}^*$ matrix can be achieved through the following operation:

$$\frac{\mathbf{dY}}{\mathbf{dX}} = \mathbf{C_Y} \frac{\mathbf{dY}^*}{\mathbf{dX}^*} \mathbf{C_X} \tag{14}$$

where $\mathbf{C_Y}$ is a $[Y \times Y^*]$ matrix with each row consisting of zeros and a single identity matrix in the column corresponding to the row of $\mathbf{dY}^*/\mathbf{dX}^*$ that is to be extracted; and where $\mathbf{C_X}$ is a $[X^* \times X]$ matrix with each column consisting of zeros and a single identity matrix in the row corresponding to the column of $\mathbf{dY}^*/\mathbf{dX}^*$ that is to be extracted.

Appropriately substituting all the values in Tab 2 and Tab 3 into Eqn 6 - 7, 9-10 and 12-14 the desired system Jacobian $\mathbf{dY}/\mathbf{dX}$ at $\mathbf{X_0}, \mathbf{Y_0}$ is determined. Due to space constraints it is not shown here.

TABLE 4. PARTIAL DERIVATIVES ASSOCIATED WITH THE COUPLING ENERGY AND MATTER FLOWS

Exchange	Descriptive Equations	Associated Partial Derivatives	Non-Zero elements
C	$\mathbf{I_L} = \mathbf{I_{L_0}} + \mathbf{C_{L\mathcal{P}}}\mathbf{I_{\mathcal{P}}} + \mathbf{C_{LF}}\mathbf{I_F} + \mathbf{C_{LD}}\mathbf{I_D}$ $\mathbf{V_F} = \mathbf{C_{FL}}\mathbf{V_L}$	$\frac{\partial \mathbf{X}_{\mathcal{E}}}{\partial \mathbf{Y}_{\mathcal{W}}}$	$\frac{\partial \mathbf{I_L}}{\partial \mathbf{I_{\mathcal{P}}}} = \mathbf{C_{L\mathcal{P}}}$ $\frac{\partial \mathbf{I_L}}{\partial \mathbf{I_F}} = \mathbf{C_{LF}}$
		$\frac{\partial \mathbf{X}_{\mathcal{W}}}{\partial \mathbf{Y}_{\mathcal{E}}}$	$\frac{\partial \mathbf{V_F}}{\partial \mathbf{V_L}} = \mathbf{C_{FL}}$
J	$\mathbf{I_L} = \mathbf{I_{L_0}} + \mathbf{C_{L\mathcal{P}}}\mathbf{I_{\mathcal{P}}} + \mathbf{C_{LF}}\mathbf{I_F} + \mathbf{C_{LD}}\mathbf{I_D}$ $\mathbf{V_D} = \mathbf{C_{DL}}\mathbf{V_L}$	$\frac{\partial \mathbf{X}_{\mathcal{E}}}{\partial \mathbf{Y}_{\mathcal{WW}}}$	$\frac{\partial \mathbf{I_L}}{\partial \mathbf{I_D}} = \mathbf{C_{LD}}$
		$\frac{\partial \mathbf{X}_{\mathcal{WW}}}{\partial \mathbf{Y}_{\mathcal{E}}}$	–
N	$\mathbf{Q_D^{rec1}} = \mathbf{C_{DF}}\mathbf{Q_F}$	$\frac{\partial \mathbf{X}_{\mathcal{W}}}{\partial \mathbf{Y}_{\mathcal{WW}}}$	–
		$\frac{\partial \mathbf{X}_{\mathcal{WW}}}{\partial \mathbf{Y}_{\mathcal{W}}}$	$\frac{\partial \mathbf{Q_D^{rec1}}}{\partial \mathbf{Q_F}} = \mathbf{C_{DF}}$
Q/P	$\mathbf{Q_D} = \mathbf{C_{DJ}}\mathbf{Q_J} = \mathbf{C_{DJ}}\mathbf{B_J^\dagger}\mathbf{Q_{\mathcal{P}}}$	$\frac{\partial \mathbf{X}_{\mathcal{W}}}{\partial \mathbf{Y}_{\mathcal{WW}}}$	–
		$\frac{\partial \mathbf{X}_{\mathcal{WW}}}{\partial \mathbf{Y}_{\mathcal{W}}}$	$\frac{\partial \mathbf{Q_D}}{\partial \mathbf{Q_{\mathcal{P}}}} = \mathbf{C_{DJ}}\mathbf{B_J^\dagger}$

7 SENSITIVITY ANALYSIS

The derived Jacobian can be used to ascertain the effects of changes in the independent variables **X** such as electric current demand, water demand, and permissible cooling water temperature increment on the dependent variables **Y**. As an illustration, consider a 10 % increase in demand for water and electric current from the base case described in Section 2 and used throughout this work. The vector $\Delta\mathbf{X}$ in this case is given by:

$$\Delta\mathbf{X} = [\mathbf{0_W}, \mathbf{0_W}, \mathbf{0_G}, \Delta\mathbf{I_{L0}}, \mathbf{0_G}, \Delta\mathbf{Q_J}, \mathbf{0_E}, \mathbf{0_S}]^\dagger \quad (15)$$

where

$$\Delta\mathbf{I_{L0}} = [0.05, 0.01, 0.01, 0.03, 0.01, 0.004, 0.006, 0.01, 0.01]^\dagger$$
$$\Delta\mathbf{Q_J} = [0.6]$$

and where $\mathbf{0_E}, \mathbf{0_G}, \mathbf{0_S}, \mathbf{0_W}$ represent zero column vectors of appropriate length. $\Delta\mathbf{Q_J}$ is in m^3/s while $\Delta\mathbf{I_{L0}}$ is per-unit current. Multiplication of the determined Jacobian by this $\Delta\mathbf{X}$ yields an approximation of $\Delta\mathbf{Y}$ shown in Tab. 5 below.

As expected, the increases in demand for water and electricty result in increases in water withdrawals, fuel consumption and brine output and in reductions in voltage and demand node pressure. The percentage changes are calculated with reference to values of **Y** determined for the base case in previous work [21, 22].

TABLE 5. SENSITIVITY ANALYSIS RESULTS

	Change	% Change
$\Delta\mathbf{Q_G^{in}}$		
$\Delta\dot{\mathbf{M}}_\mathbf{G}$	$[0.0005, 0, 0, 0, 0.059]$	$[\%, 0, 0, 0\%]$
$\Delta\mathbf{V_L}$	$[-0.0017, -0.0015, -0.0038, -0.0062, -0.0060, -0.0035, -0.0022, -0.0028, -0.0069]^\dagger p.u$	$[-0.17\%, -0.15\%, -0.36\%, -0.59\%, -0.57\%, -0.33\%, -0.21\%, -0.27\%, -0.51\%]^\dagger$
$\Delta\mathbf{Q_G^{out}}$		
$\Delta\mathbf{Q_F^*}$	$[0.2, 0.2, 0.5719]^\dagger m^3s^{-1}$	$[8.7\%, 8.7\%, 12\%]^\dagger$
$\Delta\mathbf{Q_{Jl}}$	$0.0002 m^3s^{-1}$	0.02%
$\Delta\mathbf{Q_F^{brine}}$	$[0, 0, 0.3719]^\dagger m^3s^{-1}$	$[na, na, 15.3\%]^\dagger$
$\Delta\mathbf{P_J}$	$-432Pa$	−0.04%
$\Delta\mathbf{Q_D^{disp}}$	$0.6 m^3s^{-1}$	10%

8 CONCLUSIONS AND FUTURE WORK

This work has demonstrated the derivation of the Jacobian of combined engineering systems model of the electricity, engineered water and wastewater systems and subsequently its application for sensitivity analysis. In future work efforts will be made to explicate the generalized structure of this Jacobian.

NOMENCLATURE

Γ_G Induced photocurrents at locations of solar photovoltaic installations

λ_G Wind speed at locations of wind farms

π_W Osmotic pressure of water sources

$\mathcal{H}_G$ Lower heating value of fuel used at generators

$\mathcal{H}_G^{flue}$ Specific sensible heat content of generator flue gas

$\mathbf{I_D}$ RMS Current drawn by wastewater treatment facilities

$\mathbf{I_F}$ RMS Current drawn by water treatment plants

$\mathbf{I_G}$ RMS Current supplied by generators

$\mathbf{I_L}$ RMS Current drawn by all electrical load nodes

$\mathbf{I_{L_0}}$ RMS Current drawn by all electrical nodes excluding current for water system purposes

$\mathbf{I_{L_W}}$ RMS Current drawn by all electrical nodes for water and wastewater system purposes

$\mathbf{I_N}$ RMS Current drawn by pipes and pumps in non-potable recycled wastewater distribution network

$\mathbf{I}_{\mathcal{P}}$ RMS Current drawn by pumps in water distribution network

$\dot{\mathbf{M}}_\mathbf{G}$ Fuel consumption of generators

$\dot{\mathcal{M}}_\mathbf{G}$ Process steam flow rate in thermal generators

$\mathbf{P_D}$ Pressures imposed by wastewater treatment plants on non-potable recycled wastewater distribution network

$\mathbf{P_F}$ Pressures imposed by water treatment plants on water distribution network

$\mathbf{P_J}$ Pressures at water demand nodes

$\mathbf{P_J}$ Pressures at non-potable recycled wastewater demand nodes

$\mathbf{P_W}$ Pressures of water sources

$\mathbf{Q_D}$ Throughput of wastewater treatment facilities

$\mathbf{Q_D}^{disp}$ Disposable effluent production rate at wastewater treatment facilities

$\mathbf{Q_D}^{rec}$ Non-potable recycled wastewater production rate at wastewater treatment facilities

$\mathbf{Q_E}$ Water demand rate at non-potable recycled wastewater demand nodes

$\mathbf{Q_{El}}$ Water leakage rate at non-potable recycled wastewater demand nodes

$\mathbf{Q_F}$ Water supplied to distribution network by water treatment and desalination plants

$\mathbf{Q_F^*}$ Water withdrawn by water treatment and desalination plants

$\mathbf{Q_F}^{brine}$ Brine produced by water treatment and desalination plants

$\mathbf{Q_G}^{in}$ Water withdrawal rate by generators

$\mathbf{Q_G}^{out}$ Generator effluent flow rate

$\mathbf{Q_J}$ Water demand rate at demand nodes

$\mathbf{Q_{Jl}}$ Water leakage rate at demand nodes

$\mathbf{Q_N}$ Water flow rate through pipes in non-potable recycled wastewater network

$\mathbf{Q}_{\mathcal{P}}$ Water flow rate through pipes

$\mathbf{Q_W}$ Water withdrawal rate from sources

$\mathbf{Q_S}^{evap}$ Water evaporation rate from water storage units

ΔT_W Permissible temperature increases of water sources

$\mathbf{V_F}$ RMS Voltages applied to water treatment plants

$\mathbf{V_G}$ RMS Voltage at generators

$\mathbf{V_L}$ RMS Voltage at electrical loads

$\mathbf{V_N}$ RMS Voltages applied to pumps in non-potable recycled wastewater distribution network

$\mathbf{V}_{\mathcal{P}}$ RMS Voltages applied to pumps in water distribution network

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Appendix A: SYSTEM EQUATIONS

The descriptive equations for the electricity, water and wastewater systems, previously derived in [21, 22] are shown in Eqns 16, 17 and 18 respectively.

$$
\begin{aligned}
\mathbf{Q_G}^{in} &= \mathcal{F}_{\mathbf{G_1}}(\mathbf{I_G}, \mathbf{C_{GW}}\Delta\mathbf{T_W}) \\
\mathbf{Q_G}^{out} &= \mathbf{Q_G}^{in} - \mathbf{Q_G}^{evap} \\
\dot{\mathbf{M}}_\mathbf{G} &= \mathcal{F}_{\mathbf{G_2}}(\mathbf{I_G}, \mathcal{H}_G) \\
\mathbf{V_L} &= -\mathbf{A_1}\mathbf{I_L} + \mathbf{A_2}\mathbf{V_G} \\
\mathbf{I_G} &= \mathbf{A_3}\mathbf{I_L} + \mathbf{A_4}\mathbf{V_G} \\
\mathbf{V_G} &= \mathbf{K_1} + \mathbf{K_2}\mathbf{C_{GW}}\mathbf{P_W} + \mathbf{K_3}\lambda_G + \mathbf{K_4}\Gamma_G - \mathbf{K_5}\mathbf{I_G}
\end{aligned}
\tag{16}
$$

where $\mathbf{K_1}, \mathbf{K_2}, \mathbf{K_3}, \mathbf{K_4}, \mathbf{K_5}$ are $[G \times G]$ diagonal matrices consisting of machine constants used in models of various types of generators; $\mathcal{F}_{\mathbf{G_1}}$ and $\mathcal{F}_{\mathbf{G_2}}$ are functions which include polynomial regressions of portions of the steam tables; $\mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3}, \mathbf{A_4}$ are appropriately sized matrices derived from linear combinations of the edge-node incidence matrices for the electricity supply system and a diagonal impedance matrix; and $\mathbf{C_{GW}}$ is a binary coupling matrix between generators and water sources.

$$
\begin{bmatrix}
-\beta_J(\mathbf{P_J}) & 0 & \mathbb{1}_\mathbf{J} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbb{1}_\mathbf{J} & -\mathbf{B_J}^\dagger & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{B_J} & 0 & 0 & -\mathbf{R}_\mathcal{P}(\mathbf{Q}_\mathcal{P}) & 0 & 0 & \mathbf{B_F} & 0 & 0 & 0 \\
0 & \mathbb{1}_\mathcal{P} & 0 & -\mathbf{K}_\mathcal{P}\Psi_\mathcal{P}(\mathbf{Q}_\mathcal{P}) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{B_F}^\dagger & -\mathbb{1}_\mathbf{F} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbf{K_6} & -\mathbb{1}_\mathbf{F} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbf{K_9} & 0 & -\mathbb{1}_\mathbf{F} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbf{Z_F} & 0 & 0 & -\mathbb{1}_\mathbf{F} & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbf{RR_F} & 0 & 0 & 0 & -\mathbb{1}_\mathbf{F} & 0 \\
0 & 0 & 0 & 0 & \mathbb{1}_\mathbf{F} & 0 & 0 & 0 & \mathbb{1}_\mathbf{F} & -\mathbb{1}_\mathbf{F}
\end{bmatrix}
\begin{bmatrix}
\mathbf{P_J} \\ \mathbf{I}_\mathcal{P} \\ \mathbf{Q}_{\mathbf{J_l}} \\ \mathbf{Q}_\mathcal{P} \\ \mathbf{Q_F} \\ \mathbf{I_F} \\ \mathbf{P_F} \\ \Delta\mathbf{H_F} \\ \mathbf{Q_F^{brine}} \\ \mathbf{Q_F^*}
\end{bmatrix}
+
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
\mathbb{1}_\mathbf{J} & \mathbf{C_{JS}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbf{K_8}\mathbf{C_{WF}}^\dagger & 0 & \mathbf{K_7} \\
0 & 0 & 0 & \mathbf{C_{WF}}^\dagger & \mathbf{K_{10}} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{Q_J} \\ \mathbf{Q_S}^{evap} \\ \Pi_\mathbf{W} \\ \mathbf{P_W} \\ \mathbf{V_F}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
\tag{17}
$$

where $\mathbf{B_F}$ and $\mathbf{B_J}$ are $[P \times F]$ and $[P \times J]$ edge-node incidence matrices for the water distribution system; $\mathbf{K_6}, \mathbf{K_7}, \mathbf{K_8}, \mathbf{K_9}, \mathbf{K_{10}}, \mathbf{Z_F}, \mathbf{RR_F}$ are $[F \times F]$ diagonal matrices consisting of machine constants used in models of various types of water treatment plants; $\mathbf{K}_\mathcal{P}, \mathbf{R}_\mathcal{P}, \Psi_\mathcal{P}$ are $[P \times P]$ diagonal matrices consisting of various machine constants used in models of pipes and pumps, the latter two of which are functions of $\mathbf{Q}_\mathcal{P}$; and $\mathbf{C_{WF}}$ is a binary coupling matrix between water sources and water treatment plants.

$$
\begin{bmatrix}
-\beta_E(\mathbf{P_E}) & 0 & \mathbb{1}_\mathbf{E} & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbb{1}_\mathbf{E} & -\mathbf{B_E}^\dagger & 0 & 0 & 0 \\
\mathbf{B_E} & 0 & 0 & -\mathbf{R_N}(\mathbf{Q_N}) & 0 & 0 & 0 \\
0 & \mathbb{1}_\mathbf{N} & 0 & -\mathbf{K_N}\Psi_N(\mathbf{Q_N}) & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbf{B_D}^\dagger & -\mathbb{1}_\mathbf{D} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\mathbb{1}_\mathbf{D} & 0 \\
0 & 0 & 0 & 0 & -\mathbb{1}_\mathbf{D} & 0 & -\mathbb{1}_\mathbf{D}
\end{bmatrix}
\begin{bmatrix}
\mathbf{P_E} \\ \mathbf{I_N} \\ \mathbf{Q}_{\mathbf{E_l}} \\ \mathbf{Q_N} \\ \mathbf{Q_D^{rec2}} \\ \mathbf{I_D} \\ \mathbf{Q_D^{disp}}
\end{bmatrix}
+
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & \mathbb{1}_\mathbf{E} & 0 \\
0 & 0 & 0 & \mathbf{B_D} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\mathbf{K_D} & 0 & 0 & 0 \\
\mathbb{1}_\mathbf{D} & -\mathbb{1}_\mathbf{D} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{Q_D} \\ \mathbf{Q_D}^{rec1} \\ \mathbf{Q_E} \\ \mathbf{P_D}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
\tag{18}
$$

where $\mathbf{B_D}$ and $\mathbf{B_E}$ are $[N \times D]$ and $[N \times E]$ edge-node incidence matrices for the recycled wastewater distribution system; $\mathbf{K_N}, \mathbf{R_N}, \Psi_\mathbf{N}$ are $[N \times N]$ diagonal matrices consisting of various machine constants used in models of pipes and pumps, the latter two of which are functions of $\mathbf{Q_N}$; and $\mathbf{K_D}$ is a $[D \times D]$ diagonal matrix consisting of specific current requirements for the wastewater treatment plants.