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An Axiomatic Design Approach to Non-Assembled Production Path Enumeration in Reconfigurable Manufacturing Systems

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***Abstract*—In recent years, many design approaches have been developed for automated manufacturing systems in the fields of reconfigurable manufacturing systems (RMS), holonic manufacturing systems (HMS), and multi-agent systems (MAS). One of the principle reasons for these developments has been to enhance the recongurability of a manufacturing system; allowing it to readily adapt to changes over time. However, to date, recongurability assessment has been limited. Hence, the efficacy of these design approaches remains quantitatively inconclusive. More recently, a systematic approach to reconfigurability measurement based upon the concepts of reconfiguration “potential” and reconfiguration “ease” has been developed using axiomatic design and design structure matrices respectively. The measures of reconfiguration potential called production degrees of freedom were specifically used to calculate production paths of a product line through a reconfigurable manufacturing system. This paper rearticulates the previous scalar-based calculation in terms of a axiomatic design matrix-based development founded in graph theory for three major benefits. First, this approach establishes a link between the RMS literature and graph theory where path enumeration has long been associated with network reliability and resilience. Second, the approach bases its measures strictly on the evolving system architecture variables in both function and form. In so doing, it roots itself in the established engineering design concept of the axiomatic design knowledge base for large flexible systems. Finally, the formulaic expressions lend themselves to significant computational savings.**

***Index Terms*—reconfigurability, reconfiguration processes, reconfigurable manufacturing systems, holonic manufacturing systems, multi-agent systems, graph theory**

I. Introduction

In the 1990s, manufacturing became increasingly characterized by continually evolving and ever more competitive marketplaces. The effective implementation of lean manufacturing principles, had freed excess capacity in many instances, and thus gave consumers greater inuence over the quality, quantity and variety of products [1], [2]. In order to stay competitive, manufacturing firms had to respond with a high variety products of increasingly short product life cycle [3], [4]. These dual requirements of mass-customisation and short product life cycles cause a multi-dimensional engineering management problem which enterprises have to find ways to address. One particularly pertinent problem is the need to quickly and incrementally adjust production capacity and capability. As the continually growing variety of products are introduced, ramped up, phased out, and finally made obsolete, shop floors have to find efficient ways to reallocate the capabilities of production resources to the product variants that need them most. The realization of these incremental changes is not just a tooling and fixturing setup problem. It also requires extensive change in control code and production planning systems. The problem is further exacerbated because investment into these production systems is rationalized on the basis that they produce specific products at a certain throughput to yield the desired return.

To fulfill the needs of enterprises with extensive automation, reconfigurable manufacturing systems have been proposed as a set of possible solutions [5]. They are defined as:

Definition I.1. Reconfigurable Manufacturing System [6]: [A System] designed at the outset for rapid change in structure, as well as in hardware and software components, in order to quickly adjust production capacity and functionality within a part family in response to sudden changes in market or regulatory requirements.

Over the last decade, many technologies and design approaches have been developed to enable reconfigurability in manufacturing systems [7] [8]. These have included modular machine tools [9]–[14], distributed automation [15], [16], multi-agent systems [17]–[19] and holonic manufacturing systems [2], [20]–[23]. While initial efforts to assess these design approaches have been reported [24]–[27], numerous authors have identied the need for a holistic recongurability measurement process [28]–[34]. Hence, it remains inconclusive whether the resulting designs achieve their intended level of reconfigurability. Furthermore, the absence of reconfigurability measurement has prevented the use of iterative design processes that may gradually achieve optimal results. Instead, one-off design approaches that do not build upon previous work are often investigated.

This paper builds upon a previously developed integrated reconfigurability measurement process [34]–[36] based upon the complementary principles or reconfiguration potential [37],

[38], and reconfiguration ease [39]. It makes three specific contributions. First, it rearticulates the previous scalar-based calculation in terms of an axiomatic design matrix-based development founded in graph theory. This establishes the link between the RMS literature and graph theory where path enumeration has long been associated with network reliability and resilience. Second, the approach bases its measures strictly on the evolving system architecture variables in both function and form. In so doing, it roots itself in an established engineering design concept: the axiomatic design knowledge base for large flexible systems. This facilitates the original purpose of the measurement process, namely an iterative engineering design process based upon quantitative assessment. Finally, the formulaic expressions lend themselves to significant computational savings for straightforward implementation in a design environment. The approach has previously been demonstrated on reconfigurable transportation systems [40], [41] as a subset of the more complex manufacturing problem.

The paper follows a six part discussion. Section II orients the discussion in terms of the two foundations of the work: graph theory and axiomatic design. Next, Section III continues the background to recall the previously developed measures of production degrees of freedom with new matrix-based additions. Section IV then utilizes these measures to enumerate production paths. The developments are then demonstrated on an illustrative example in Section V. Finally, Section VI concludes the work with a thorough discussion of the utility of the developments. Prior to proceeding, this paper restricts its discussion to the shop-floor activities of automated manufacturing system as defined in Levels 0-3 of ISA-S95 [42]. Furthermore, it defines reconfigurability as

Definition I.2. Reconfigurability [35]: The ability to add, remove and/or rearrange in a timely and cost-effective manner the components and functions of a system which can result in a desired set of alternate congurations.

II. Background

This section summarizes the methodological developments found in graph theory and axiomatic design in order to provide a foundation for the description of production degrees of freedom in th next section. Section II-A gives a brief introduction to graph theory while Section II-B introduces the application of axiomatic design for large flexible systems to production systems. Finally, Section II-C describes the key characteristics of mechanical degrees of freedom upon which the work is based.

A. Graph Theory Introduction

Graph theory is a long established field of mathematics with applications in many fields of science and engineering where artifacts are transported between physical locations [43]–[45]. A number of definitions from this field are introduced for later in the development.

Definition II.1. A graph [44]: $G = \{V, E\}$, consists of a collection of nodes V and a collection of edges E. Each edge $e \in E$ is said to join two nodes which are called its end points. If e joins $v_1, v_2 \in V$, we write $e = \langle v_1, v_2\rangle$. Nodes v_1 and v_2, in this case, are said to be adjacent. Edge e is said to be incident with nodes v_1 and v_2 respectively.

Definition II.2. A directed graph (digraph) [44]: D, consists of a collection of nodes V and a collection of arcs A, for which $D = V, A$. Each arc $a = \langle v_1, v_2\rangle$ is said to join node $v_1 \in V$ to another (not necessarily distinct) node v_2. Vertex v_1 is called the tail of a, whereas v_2 is its head.

Definition II.3. Adjacency matrix [44]: A, is binary and of size $\sigma(V) \times \sigma(V)$ and its elements are given by

$$A(i, j) = \begin{cases} 1 & \text{if } \langle v_i, v_j\rangle \text{ exists} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where the operator $\sigma()$ gives the size of a set. Interestingly, $A^N(i, j)$ represents the number of n-step paths between an origin i and destination j [43].

Definition II.4. Incidence matrix [44]: M of size $\sigma(V) \times \sigma(A)$ is given by:

$$M(i, j) = \begin{cases} -1 & \text{if vertex } v_i \text{ is the head of arc } a_j \\ 1 & \text{if vertex } v_i \text{ is the tail of arc } a_j \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

While graph theory for decades has presented a useful abstraction across many fields including the related field of transportation, it has limitations from an engineering design and systems engineering perspective [46]. The main challenge is finding robust approaches to linking nodes and arcs to physical variables [43]. In the context of this work, the above definitions focus on the abstract form of the system, and less so the production functions. Furthermore, how the function is realized into form variables is not explicitly stated. Unless generalized, such graph theoretic approaches are likely to have limitations in production systems of heterogeneous function and form. Furthermore, because the system function and its realizing form have been abstracted away, such approaches may not straightforwardly lend themselves to active control solutions that implement reconfigurable manufacturing system architectures.

B. Axiomatic Design for Large Flexible Systems

In contrast, axiomatic design of large flexible systems provides a natural engineering design description of production systems. Suh [47] defines large flexible systems as systems with many functional requirements that not only evolve over time, but also can be fulfilled by one or more design parameters [47]. In production systems, the high level design parameters are taken as the set of production resources. **DP**={Production resources}. These resources $R = M \cup B \cup H$ may be classified into value adding machines $M = \{m_1 \ldots m_{\sigma(M)}\}$, independent buffers $B = \{b_1 \ldots b_{\sigma(B)}\}$, and material handlers $H = \{h_1 \ldots h_{\sigma(H)}\}$. The set of buffers $B_S = M \cup B$ is also introduced for later simplicity. Similarly, the high level functional requirements are taken as a set of production processes. **FR**={Production Processes}. These are formally classified into their transformation and transportation varieties $P = P_\mu \cup P_\eta$, and defined as:

Definition II.5. Transformation Process [37]: A machine-independent, manufacturing technology-independent process $p_{\mu j} \in P_\mu = \{p_{\mu j} \ldots p_{\mu\sigma(P_\mu)}\}$ that transforms raw material or work-in-progress to a more final form.

Definition II.6. Transportation Process [37]: A material-handler - independent process $p_{\eta u} \in P_\eta = \{p_{\eta u} \ldots p_{\eta\sigma(P_\eta)}\}$ that transports raw material, work-in-progress, or final goods from buffer b_{sy_1} to b_{sy_2}. There are $\sigma^2(B_S)$ such processes of which $\sigma(B_S)$ are "null" processes where no motion occurs. Furthermore, the convention of indices $u = \sigma(B_S)(y_1-1)+y_2$ is adopted.

Given that this work finds later application in engineering design, these production processes and resources may be related through the use of the axiomatic design equation for large flexible systems [47]. In the case of production systems with a physically allocated [15] distributed control system:

$$P = J_S \odot R \tag{3}$$

where J_S is a binary matrix called a "knowledge base", and $\odot$ is "matrix boolean multiplication" [37]. In other words, the production system knowledge base itself forms a bipartite graph [44] which maps the set of production processes to production resources. Production systems with centralized controllers have been previously addressed [37].

C. Mechanical & Production Systems: An Analogy

Finally, the notion of production degrees of freedom (DOF) is based upon a strong analogy to mechanical systems which has been previously drawn elsewhere and is summarized in Table I.

Table I: Mechanical & Production System Analogy [37]

	Mechanical	**Production**
State:	Continuous	Discrete
State Change:	Leads to Motion	Leads to Production
State Evolution:	Time Driven	Event Driven
Function:	Rotation,Translation	Transformation,Transportation
Coordinates:	Dimension & Link	Process & Resource

The importance of the degree of freedom analogy is fundamental. In mechanical systems, DOFs form the basis vector upon which the entire system behavior is described and evolves. Here, the production DOFs serve the same purpose but given their discrete nature also allow for a complete description of the variables of a system's architecture. From the reconfigurability definition, reconfiguration potential can be interpreted in two ways: 1.) the ability of a system to simultaneously hold multiple desired configurations 2.) the ability of a system to change to a desired set of alternate congurations. In either case, a DOF approach appears as a suitable starting point for a reconfiguration potential measure. Such a number would indicate the systems multiple configurations. It would also change as the system underwent successive reconfigurations. In summary, a DOF approach is suitable for measuring reconfiguration potential because it describes the elements of a system and how they may be combined into numerous productive configurations [37].

III. Production Degrees of Freedom

This section continues the background to recall the previously developed measures of production degrees of freedom [37]. Along the way, matrix-based developments are introduced to replace scalar-based work [38]. Given the abstract nature of the mathematical treatment, the interested reader is referred to prior work for illustrative examples and case studies [34], [37], [38]. The discussion proceeds in two parts. Sequence independent production degrees of freedom are first introduced followed by sequence dependent measures.

A. Sequence-Indepedent Production Degrees of Freedom

The heart of the production degrees of freedom concept rests in the realization that an event $e_{wv} \in E$ (in the discrete-event system sense) [48] can be defined for each feasible combination of production process p_w and resource r_v. Later on, reconfigurations can add or remove these events or potentially reallocate a process to a resource.

Definition III.1. Production System Knowledge Base [37]: A binary matrix J_S of size $\sigma(P) \times \sigma(R)$ whose element $J_S(w, v) \in \{0, 1\}$ is equal to one when event e_{wv} exists.

The development of production degrees of freedom continues with the introduction of a number constraints as is found in mechanical degrees of freedom. Here, the constraints are discrete and can apply in the operational time frame so as to eliminate events from the event set. These constraints are said to be *scleronomic* as they are independent of event sequence. Such constraints can arise from any phenomenon that reduces the capabilities of a production system e.g. resource breakdowns, inflexibly implemented production processes and their control.

Definition III.2. Production System Scleronomic Constraints Matrix [37]: A binary matrix K_S of size $\sigma(P) \times \sigma(R)$ whose element $K_S(w, v) \in \{0, 1\}$ is equal to one when a constraint eliminates event e_{wv} from the event set.

From these definitions of J_S and K_S, follows the definition of sequence-indepedent production degrees of freedom.

Definition III.3. Sequence-Independent Production Degrees of Freedom [37]: The set of independent production events E_S that completely defines the available production processes in a production system. Their number is given by:

$$DOF_S = \sigma(\mathcal{E}_S) = \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(R)} [J_S \ominus K_S](w, v) \tag{4}$$

where $A \ominus B$ operation is "boolean subtraction". Alternatively, $A \ominus B$ is equivalent to $A \cdot \bar{B}$. Note that the boolean "AND" $\cdot$ is equivalent to the hadamard product, and $\bar{B}$ = not(B). In matrix form, Equation 4 can be rewritten in terms of the Frobenius inner product [49].

$$DOF_S = \langle J_S, \bar{K}_S \rangle_F = tr(J_S^T \bar{K}_S) \tag{5}$$

These sequence-independent production degrees of freedom may also be classified into their transformational (DOF_M) and transportational (DOF_H) variants as shown in Table II. In this case, it follows that [37]:

$$J_S = \left[\begin{array}{c|c} J_M & \mathbf{0} \\ \hline \multicolumn{2}{c}{J_{\bar{H}}} \end{array}\right] \tag{6}$$

$$K_S = \left[\begin{array}{c|c} K_M & \mathbf{1} \\ \hline \multicolumn{2}{c}{K_{\bar{H}}} \end{array}\right] \tag{7}$$

Table II: Types of Sequence-Independent Production Degree of Freedom Measures [37].

Measure	Process Element	Resource Element	Knowledge Base	Constraint Matrix	Measure Function
DOF_M	$p_{\mu j}$	m_k	J_M	K_M	$\langle J_M, \bar{K}_M \rangle_F$
DOF_H	$p_{\eta u}$	r_v	J_H	K_H	$\langle J_H, \bar{K}_H \rangle_F$
$DOF_{\bar{H}}$	$P_{\gamma \varrho}$	r_v	$J_{\bar{H}}$	$K_{\bar{H}}$	$\langle J_{H\gamma}, \bar{K}_{H\gamma} \rangle_F$
DOF_S	p_w	r_v	J_S	K_S	$\langle J_{S\gamma}, \bar{K}_{S\gamma} \rangle_F$

One notable difference is the measure for refined transportation degrees of freedom $DOF_{\bar{H}}$ which distinguishes events in the transportation system on the basis of the holding processes of each resource's fixture or end-effector.

Definition III.4. Holding Process [37]:A material-handler and end-effector-independent process $p_{\varphi g}$ that holds raw material, work-in-progress, or final products during the transportation from one buffer to another.

In this case, a knowledge base J_φ and constraint matrix K_φ of size $\sigma(P_\varphi) \times \sigma(R)$ are constructed to capture the holding capabilities of the production system. Then, the refined scleronomic transportation knowledge base $J_{\bar{H}}$ and constraints matrix $K_{\bar{H}}$ are formed using the kronecker tensor product and column selection.

$$\begin{aligned} J_{\bar{H}} &= \left[J_\varphi \otimes \mathbf{1}^{\sigma(P_\eta)}\right] \cdot \left[\mathbf{1}^{\sigma(P_\varphi} \otimes J_{\bar{H}}\right] \\ K_{\bar{H}} &= \left[K_\varphi \otimes \mathbf{1}^{\sigma(P_\eta)}\right] \cdot \left[\mathbf{1}^{\sigma(P_\varphi} \otimes K_{\bar{H}}\right] \end{aligned} \tag{8}$$

where $\otimes$ is the Kronecker tensor product and $\mathbf{1}^n$ is a ones vector of length n.

Intuitively, the sequence-independent production degree of freedom measure the number of ways that all of the variable transformation processes may be executed. They provide a flexible expression of production system capabilities in the design and operational phases. From an axiomatic design perspective, the usage of knowledge bases facilitates further detailed engineering design. The constraints matrix captures the potential for resource breakdowns and inflexibly implemented processes either physically or informatically in associated control structures. Additionally, from a graph theory perspective, the knowledge base forms a bipartite graph which may experience edge elimination during operation. Finally, the mathematical form of the production DOF measures match the form of the mechanical DOF measures. The conceptual ties to graph theory and mechanical degrees of freedom suggest that useful results from these fields can potentially be applied to production systems for the first time [37].

B. Sequence Dependent Production Degrees of Freedom

The previous subsection recalled the development of production degrees of freedom as sequence independent. A production system, however, has constraints that introduce dependencies in the sequence of events. A new measure is required for the sequence dependent capabilities of the production system [37].

Definition III.5. Sequence Dependent Production Degrees of Freedom [37]: The set of independent production strings $\mathcal{Z}$ of length 2 that completely describe the production system language.

In other words, the production system language $\mathcal{L}$ can be described equally well in terms of the Kleene closure [48] of the sequence-independent and sequence-dependent production degrees of freedom.

$$\mathcal{L} = \mathcal{E}^* = \mathcal{Z}^* \tag{9}$$

The calculation of sequence dependent production degrees of freedom closely follows the approach in the previous section. The strings $z_{\psi_1 \psi_2} = e_{w_1 v_1} e_{w_2 v_2} \in Z$ can be captured succinctly in a *rheonomic* (i.e sequence-dependent) production system knowledge base.

Definition III.6. Rheonomic production system knowledge base: A square binary matrix J_ρ of size $\sigma(P)\sigma(R) \times \sigma(P)\sigma(R)$ whose element $J(\psi_1, \psi_2) \in \{0, 1\}$ are equal to one when string $z_{\varrho,\psi}$ exists. It may be calculated directly as

$$J_\rho = \left[J_S \cdot \bar{K}_S\right]^V \left[J_S \cdot \bar{K}_S\right]^{VT} \tag{10}$$

where the A^V operation is shorthand for vectorization $vec()$ commonly implemented in MATLAB with the (:) operator [37]. Here, the scleronomic production degrees of freedom are explicitly treated as a basis vector – as would typically be done with mechanical degrees of freedom. Interestingly, the J_ρ also strongly resembles an adjacency matrix where the degrees of freedom are treated as mutually connected nodes.

The development proceeds with the introduction of rheonomic constraints.

Definition III.7. Rheonomic Production Constraints Matrix K_ρ: a square binary constraints matrix of size $\sigma(P)\sigma(R) \times \sigma(P)\sigma(R)$ whose elements $K(\psi_1, \psi_2) \in \{0, 1\}$ are equal to one when string $z_{\psi 1 \psi 2}$ is eliminated.

Unlike its scleronomic counterpart, the rheonomic production constraints matrix has the perpetually binding constraints described in Table III. These ensure that the origin and destination of consecutive events match. Accurately keeping track of these constraints simultaneously is challenging. The final calculation of these minimal constraints is most easily implemented in a scalar fashion using FOR loops while adhering to the following relationships of indices. $\psi = \sigma(P)(v-1)+w$. $v = k \; \forall k = [1 \ldots \sigma(M)]$. $w = [\sigma(P_\eta)(g-1)+u]+j$.

Table III: Types of Sequence-DependentProduction Degree of Freedom Measures [37].

Type	Measures	Processes	Resources	Knowledge Base	Constraint Matrix	Perpetual Constraint	Measure Function
I	$DOF_{MM\rho}$	$P_\mu P_\mu$	M, M	$J_{MM\rho} = J_M \otimes J_M$	$K_{MM\rho}$	$K_1 = K_2$	$\langle J_{MM\rho}, \bar{K}_{MM\rho} \rangle_F$
II	$DOF_{MH\rho}$	$P_\mu P_\eta$	M, R	$J_{MH\rho} = J_M \otimes J_H$	$K_{MH\rho}$	$k_1 - 1 = (u_1 - 1)/\sigma(B_S)$	$\langle J_{MH\rho}, \bar{K}_{MH\rho} \rangle_F$
III	$DOF_{HM\rho}$	$P_\eta P_\mu$	R, M	$J_{HM\rho} = J_H \otimes J_M$	$K_{HM\rho}$	$k_1 - 1 = (u_1 - 1)\&\sigma(B_S)$	$\langle J_{HM\rho}, \bar{K}_{HM\rho} \rangle_F$
IV	$DOF_{HH\rho}$	$P_\eta P_\eta$	R, R	$J_{HH\rho} = J_H \otimes J_H$	$K_{HH\rho}$	$(u_1 - 1)\%\sigma(B_S) = (u_2 - 1)/\sigma(B_S)$	$\langle J_{HM\rho}, \bar{K}_{HM\rho} \rangle_F$
ALL	DOF_ρ	PP	R, R	$J_\rho = J_S \otimes J_S$	K_ρ		$\langle J_{HM\rho}, \bar{K}_{HM\rho} \rangle_F$

It follows that the number of sequence-dependent production degrees of freedom is:

$$DOF_\rho = \sigma(\mathcal{Z}) = \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} [J_\rho \ominus K_\rho](\psi_1, \psi_2) \tag{11}$$

As with scleronomic production degrees of freedom, rheonomic production degrees of freedom may be classified in terms of their transformational and transportational variants. The calculation of the four types of measures is summarized in Table III [37] and maintains an intuitive symmetry. In practice, the formation of the associated constraints matrices $K_{MM\rho}$, $K_{MH\rho}$, $K_{HM\rho}$,$K_{HH\rho}$ is an extra computational expense if K_ρ has already been formed. Instead, the associated rheonomic production degree of freedom measures can be calculated by the appropriate replacement of J_M or $J_{\bar{H}}$ with a zero matrix in Equation 6.

C. Summary of Production Degrees of Freedom

The need for these variants arises from two specific characteristics of production systems. First, transformation and transportation activities have intrinsically different values in production and therefore have to be distinguished. While the conventional wisdom is to maximize "value-adding" processes in production, this work shows that excessive elimination of transportation processes can cripple the transformation capabilities of the production system as a whole. Next, products impose particular sequences of production. Therefore, production system capabilities need to be considered not just as a loose collection of transformations and transportations but also as integrated sequences. These two differences account for the six elemental production DOF types and their two combinations [37].

The production degree of freedom measures provide a quantitative description of "reconfiguration potential" because they describe a system's capabilities and how it can be changed. Generally speaking, a reconfiguration process can be classified as: changes in the production resources, their processes or their combinations. Mathematically, production degrees of freedom would describe such a reconfiguration as [36], [37]:

$$(J_S, K_S, K_\rho) \rightarrow (J'_S, K'_S, K'_\rho) \tag{12}$$

IV. Product Degrees of Freedom

While it is important to quantify a production system's capabilities, it is even more important to assess how well these capabilities are matched to the current product line – especially if that product line is in flux due to mass-customization and short product life cycles. On this basis, one can decide if the production system requires the elimination or addition of production degrees of freedom. This sections builds upon the effort of the previous section to develop product degree of freedom measures. These measures address how well the current product line utilizes the existing production system capabilities and the number of ways that each product may be produced. To begin, the product line is simplistically modeled so that sequence-independent and sequence-dependent product degree of freedom measures may be developed later.

A. Product Modeling

Before a treatment of product degrees of freedom can be initiated, a systematic approach to describing products is required. Many product modeling approaches have been developed previously [50]. However, this treatment has a number of clear requirements not previously found in other approaches. Specically, the modeling approach has to be simultaneously comprehensive and minimalistic. A comprehensive approach would not limit the broad scope developed in the previous section. At the same time, the approach must be minimalistic so as to extract only the aspects of products most salient to production degrees of freedom. Such aspects include a description conducive to discrete events, transformation processes and holding processes. Finally, the approach must be quantitative to support the ultimate objective of developing a recongurability measure.

One prominent class of products are those that do not require assembly operations and whose production may be described as a fixed sequence. A given enterprise may have a whole product line $L = \{l_1, \ldots, l_{\sigma(L)}\}$. Each product l_i has its associated set of product events $e_{xl_i} \in E_{l_i}$ which when all are completed result in a fully manufactured product.

Definition IV.1. Product Event [38]: A specific transformation process that may be applied to a given product.

A fixed sequence non-assembled product has a unique sequence [38]:

$$z_{l_i} = e_{x_1 l_i} e_{x_2 l_i} \ldots e_{x_{\sigma(E_{l_i})} l_i} \tag{13}$$

which describes the full production of the product. This string is sufficient to represent a non-assembled product with no disjoining processes.

B. Sequence-Independent Product Degrees of Freedom

Product modeling facilitates the discussion of product degrees of freedom. Here, the interest is in determining how well the production system's capabilities match the requirements of the full product line. The sequence-indepdent product degrees of freedom are much like their production counterparts and have their transformation and transportation variants. Intuitively speaking, the method of calculation utilizes a product transformation and transportation feasibility matrices that link product events to their corresponding processes and the associated degrees of freedom are selected out of the available event set. Product transformation and transportation feasibility is defined before presenting the method of calculation.

1) Product Transformation Feasibility:

Definition IV.2. Product Transformation Degrees of Freedom [38]: The set of independent production events $\mathcal{E}_{LM}$ that completely define the available combinations of transformation process and machine that are required by the production of the product line.

A measure of product transformation degrees of freedom gives a sense of how well production capabilities are utilized. It encourages resources to be decommissioned due to non-use, new products to be added for greater return on investment, or even new processes to be added to eliminate product infeasibility. The key to their development is to link each product event (in each product i) to a unique production process with a binary map.

Definition IV.3. Product Transformation Feasibility Matrix $\Lambda_{\mu i}$ [38]: A binary matrix of size $\sigma(E_{l_i}) \times \sigma(P_\mu)$ whose value $\Lambda_{\mu i}(x, j) = 1$ if e_{xl_i} realizes transformation process $p_{\mu j}$.

2) Product Transportation Feasibility: Product transportation degrees of freedom is addressed similarly.

Definition IV.4. Product Transportation Degrees of Freedom: The set of independent product events $\mathcal{E}_{LH}$ that completely define the available combinations of transportation process and machine that can be utilized in the production of the product line.

One notable difference, however, is that products do not intrinsically require transportation processes. Transportation in production systems only occurs because of the inability of a given machine to realize all of the products events. Nevertheless, a product transportation degree of freedom measure is required to assess a product lines utilization of the transportation systems capabilities. In such a way, it encourages a thoughtful rationalization of each transportation resource. Similarly to the previous subsection, feasibility is addressed with a binary map.

Definition IV.5. Product Transportation Feasibility Matrix $\Lambda_{\gamma i}$ [38]: A binary row vector of size $1 \times \sigma(P_\gamma)$ whose value $\Lambda_{\gamma i}(g) = 1$ if product l_i can be held by holding process $p_{\gamma g}$.

3) Degree of Freedom Calculation: The transformation and transportation variants of the product feasibility matrices can be used to produce a number of "selector" matrices that are in equal in size to their corresponding scleronomic production knowledge base. Which one is used depends on the user's scope of interest be it the type of production process (e.g. transformation or transportation) or at the scale of a single product event, a product, or the whole product line. Table IV summarizes the definition and formulation of the multiple types of product selector matrices.

From these definitions, it is straightforward to assess the number of product transformation and transportation degrees of freedom.

$$DOF_{LM} = \langle \Lambda_{ML} \cdot J_M, \bar{K}_M \rangle_F \tag{14}$$

$$DOF_{LH} = \langle \Lambda_{HL} \cdot J_{\bar{H}}, \bar{K}_{\bar{H}} \rangle_F \tag{15}$$

This intuitive form of product degrees of freedom shows that the product line effectively selects out the production degrees of freedom provided by the production system. The former is ultimately a subset of the latter as a product naturally restricts the scope of a production system [38].

Table IV: Types of Product Selector Matrices

Symbol	Formula	Scope
Λ_{Mxi}	$\left[e_x^T \Lambda_{\mu i}\right]^T \mathbf{1}^{\sigma(M)T}$	Product Event – Transformation
Λ_{Mi}	$\left[\bigvee_x^{\sigma(E_L)} \Lambda_{\mu i}\right]^T \mathbf{1}^{\sigma(M)T}$	Product – Transformation
Λ_{ML}	$\left[\bigvee_i^{\sigma(L)} \bigvee_x^{\sigma(E_L)} \Lambda_{\mu i}\right]^T \mathbf{1}^{\sigma(M)T}$	Product Line – Transformation
Λ_{Hi}	$\left[\Lambda_{\gamma i} \otimes \mathbf{1}^{\sigma(P_\eta)T}\right]^T \mathbf{1}^{\sigma(R)T}$	Product – Transportation
Λ_{Hi}	$\left[\left[\bigvee_i^{\sigma(L)} \Lambda_{\gamma i}\right] \otimes \mathbf{1}^{\sigma(P_\eta)T}\right]^T \mathbf{1}^{\sigma(R)T}$	Product – Transportation
Λ_{Sxi}	$\left[\begin{array}{cc} \Lambda_{Mxi} \vert & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{Hi}} \end{array}\right]$	Product Event – Transformation & Transportation
Λ_{SMxi}	$\left[\begin{array}{cc} \Lambda_{Mxi} \vert & \mathbf{0} \\ \hline \multicolumn{2}{c}{\mathbf{0}} \end{array}\right]$	Product Event – Transformation
Λ_{Si}	$\left[\begin{array}{cc} \Lambda_{Mi} \vert & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{Hi}} \end{array}\right]$	Product – Transformation & Transportation
Λ_{SHi}	$\left[\begin{array}{cc} \mathbf{0} \vert & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{Hi}} \end{array}\right]$	Product – Transportation
Λ_{SL}	$\left[\begin{array}{cc} \Lambda_{ML} \vert & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{HL}} \end{array}\right]$	Product Line – Transformation & Transportation

C. Production Path Enumeration: Rheonomic Product Degrees of Freedom

Given the foundation set from the previous sections, the enumeration of production paths may be viewed as the calculation of rheonomic product degrees of freedom.

Definition IV.6. Rheonomic Product Degrees of Freedom [38]: The set of independent product strings Z_L that completely define all the ways that the product line may be produced.

Such a measure is important because it not only identifies the numerous production paths for a product, but it could also highlight which capabilities are required to make the product's production feasible.

The first step in the development is the translation of the product event string to an equivalent string composed of production degrees of freedom. Therefore, the string $z_{l_i} =$

$e_{1l_i}, \ldots, e_{\sigma(E_{l_i})l_1}$ can be translated into an equivalent string in transformation and transportation degrees of freedom are alternated [38]:

$$z = \begin{array}{l} e_{\mu_{j_1} m_{k_1}} [e_{\gamma_{g_1} r_{v_1}} e_{\eta_{u_1} r_{v_1}}] e_{\mu_{j_2} m_{k_2}} \\ {[e_{\gamma_{g_2} r_{v_2}} e_{\eta_{u_2} r_{v_2}}]} \ldots e_{\mu_{j_{\sigma(E_{l_i})}} m_{k_{\sigma(E_{l_i})}}} \end{array} \tag{16}$$

This string encompasses the string $z = e_{\mu_{j_1} m_{k_1}} e_{\mu_{j_2} m_{k_2}} \ldots e_{\mu_{j_{\sigma(E_{l_i})}} m_{k_{\sigma(E_{l_i})}}}$ because the holding-transportation event pair can occur at the value-adding machine that realizes the prior and subsequent transformations. In this case, the pair of events act as null processes/events. Therefore, two rheonomic knowledge bases can be formed following Equation 10. The first represents the Type II rheonomic production degree of freedom *from* product event e_{x_1} while the second represents the Type III DOF *to* product event e_{x_2}.

$$J_{MHx_1\rho} = \left[\Lambda_{SMx_1} \cdot J_{SM} \cdot \bar{K}_S M\right]^V \left[\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_S H\right]^{VT} \tag{17}$$

$$J_{HMx_2\rho} = \left[\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_S H\right]^V \left[\Lambda_{SMx_2} \cdot J_{SM} \cdot \bar{K}_S M\right]^{VT} \tag{18}$$

The rest of the formulation relies on the simple graph theory result that an adjacency matrix raised to the n^{th}-power can be used to calculate the number of n-step paths within a network. Taking into the rheonomic constraints matrices, the two rheonomic knowledge bases already represent adjacency matrices. They can then be multiplied together to give the paths between two product events.

$$A_{\rho x} = \left[J_{MH\rho x} \cdot \bar{K}_\rho\right] \left[J_{HM\rho x+1} \cdot \bar{K}_\rho\right] \tag{19}$$

This adjacency matrix can then be raised to the $\sigma(E_{l_i}) - 1$ power to account for the full product string. The final number of production paths is the sum of the elements of the resulting matrix.

$$DOF_P = \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} \left[\prod_{x}^{\sigma(E_l)-1} A_{\rho x}\right] (\psi_1, \psi_2) \tag{20}$$

V. Illustrative Example

As has been demonstrated in prior work, the "Starling Manufacturing System" is chosen as an illustrative example with two modifications [34], [37], [38]. Otherwise, the associated data remains unchanged. The assembly station has been replaced with lamination and only one shuttle transports products to various machines. Figure 1 shows the chosen configuration. For simplicity, the product line is taken as a single green bird feeder composed of a single cylinder that requires turning, lamination and painting. The associated product transformation feasibility matrix is $\Lambda_{\mu 1} = [e_1 e_2 e_3 e_4 e_7]]^T$ where e_n is an elementary row vector of appropriate size.

The degree of freedom measures follow straightforwardly. The scleronomic DOF measures in Table II evaluate to: $DOF_M = 10$, $DOF_H = 16$, $DOF_{\bar{H}} = 19$, $DOF_S = 29$. Of these DOFs, 8 and 16 can be used by the product for the DOF_{LM} and DOF_{LH} measures. The rheonomic DOF measures in Table III evaluate to: $DOF_{MM\rho} = 28$, $DOF_{MH\rho} = 33$, $DOF_{HM\rho} = 32$, $DOF_{HH\rho} = 62$, $DOF_\rho = 155$. These lead to $DOF_P = 4$ production paths which can be verified visually.

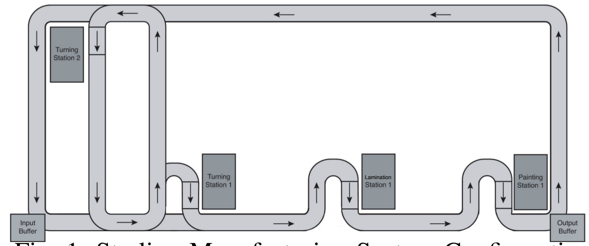

Fig. 1: Starling Manufacturing System Configuration

From a computational perspective, the most taxing activity was the generation of K_ρ which took approximately 3 seconds. Otherwise, all of the measures were straightforwardly calculated in fractions of a second on a 2.8GHz process. This represents a significant improvement over a previous method in which an assembled product with 16 events executed in 120 seconds on a 2GHz process [38]. While these figures can not be directly compared because of their applicability (i.e. assembled vs non-assembled products) and size (5 vs 16 product events), the initial evidence suggests significant computational savings. The ease of these calculations facilitates their adoption in more advanced applications.

VI. Discussion & Conclusions

In conclusion, this paper succinctly develops measures of production and product degrees of freedom and utilizes them to enumerate a product's production paths. This rearticulation utilizes matrix-based formulations that lead to significant computational savings. In the meanwhile, a strong link between axiomatic design and graph theory is established. In so doing, the paper roots itself in the established engineering design literature while also allowing for useful results from graph theory to be applied to production systems for the first time. Finally, the strong resemblance of production degrees of freedom to their mechanical counterparts suggests further potential for utility.

The usage of constraints matrices allows for any phenomenon that reduces the capabilities of a production system e.g. resource breakdowns, inflexibly implemented production processes and their control. The first of these can be used to test a production system's resilience in the face of disruption. The latter suggests that a direct link can be made between flexible production control systems (e.g. multi-agent systems) and the impact on operating revenue.

The knowledge bases themselves can be used directly as design decision variables, while the measures themselves can be iteratively optimized. This is particularly the case when assessing how well the production system's capabilities are utilized by the existing product line.

Finally, the study of sequence-dependent capabilities and production paths facilitates the rationalization of resources as a whole. Greater system resilience can be achieved with redundancy or with on-the-fly reconfiguration. Similarly, "non-value adding" transportation resources (e.g. AGVs) can demonstrate their necessary role in production paths.

References

[1] S. Goldman, R. Nagel, and K. Preiss, *Agile Competitors and Virtual Organisations*. Van Nostrand Reinhold, 1995.

[2] D. McFarlane and S. Bussmann, "Holonic manufacturing control: Rationales, developments and open issues," in *Agent-Based Manufacturing*, S. Deen, Ed. Berlin: Springer-Verlag, 2003, ch. 13, pp. 303–326.

[3] M. Mehrabi, A. Ulsoy, Y. Koren, and P. Heytler, "Trends and perspectives in flexible and reconfigurable manufacturing systems," *Journal of Intelligent Manufacturing*, vol. 13, no. 2, pp. 135–146, 2002.

[4] J. Pine, *Mass Customization: The New Frontier in Business Competition*. Boston, MA: Harvard Business School Pressed, 1993.

[5] M. Mehrabi, A. Ulsoy, and Y. Koren, "Reconfigurable manufacturing systems and their enabling technologies," *International Journal of Manufacturing Technology and Management*, vol. 1, no. 1, pp. 113–130, 2000.

[6] Y. Koren, U. Heisel, F. Jovane, T. Moriwaki, G. Pritschow, G. Ulsoy, and H. Van Brussel, "Reconfigurable manufacturing systems," *CIRP Annals - Manufacturing Technology*, vol. 48, no. 2, pp. 527–540, 1999.

[7] A. I. Dashchenko, *Reconfigurable Manufacturing Systems and Transformable Factories*. Springer, 2006.

[8] R. Setchi and N. Lagos, "Reconfigurability and reconfigurable manufacturing systems – state of the art review," in *Innovative Production Machines and Systems: Production Automation and Control State of the Art Review*. Cardiff, United Kingdom: Innovative Production Machines and Systems: Network of Excellence, 2005, ch. 5, pp. 131–143.

[9] S. Farritor and J. Zhang, "A reconfigurable robotic system to support planetary surface operations," 2001.

[10] J. Heilala and P. Voho, "Modular reconfigurable flexible final assembly systems," *Assembly Automation*, vol. 21, pp. 20–28, 2001.

[11] J. Ho and P. Ranky, "An object-oriented and flexible material handling system," *Assembly Automation*, vol. 15, no. 3, pp. 15–20, 1995.

[12] R. G. Landers, B. K. Min, and Y. Koren, "Reconfigurable machine tools," *CIRP Annals - Manufacturing Technology*, vol. 50, pp. 269–274, 2001.

[13] B. Shirinzadeh, "Flexible fixturing for workpiece positioning and constraining," *Assembly Automation*, vol. 22, pp. 112–20, 2002.

[14] W. Townsend, "The barretthand grasper-programmably flexible part handling and assembly," *Industrial Robot*, vol. 27, pp. 181–8, 2000.

[15] R. Brennan and D. Norrie, "Agents, holons and function blocks: Distributed intelligent control in manufacturing," *Journal of Applied Systems Science: Special Issue*, vol. 2, no. 1, pp. 1–19, 2001.

[16] V. Vyatkin, *IEC 61499 Function Blocks for Embedded and Distributed Control Systems*, 2006.

[17] W. Shen and D. Norrie, "'agent-based systems for intelligent manufacturing: A state-of-the-art survey," *Knowledge and Information Systems: An International Journal*, vol. 1, no. 2, pp. 129–156, 1999.

[18] W. Shen, D. Norrie, and J. Barthes, *MultiAgent Systems for Concurrent Intelligent Design and Manufacturing*. Taylor and Francis, 2000.

[19] P. Leitao, "Agent-based distributed manufacturing control: a state-of-the-art survey," *Engineering Applications of Artificial Intelligence*, vol. 22, no. 7, pp. 979–991, Oct. 2009.

[20] R. F. Babiceanu and F. F. Chen, "Development and applications of holonic manufacturing systems: A survey," *Journal of Intelligent Manufacturing*, vol. 17, pp. 111–131, 2006.

[21] A. Farid, "A review of holonic manufacturing systems literature," University of Cambridge Institute for Manufacturing, Cambridge UK, Tech. Rep., 2004.

[22] V. Marik, M. Fletcher, M. Pechoucek, O. Stepankova, H. Krautwurmova, and M. Luck, "Holons and agents: Recent developments and mutual impacts," in *Multi-Agent Systems and Applications II: Lecture Notes in Artificial Intelligence*. Springer Verlag, 2002, pp. 233–267.

[23] D. McFarlane and S. Bussmann, "Developments in holonic production planning and control." *Production Planning and Control*, vol. 11, no. 6, pp. 522–536, 2000.

[24] R. Brennan and D. Norrie, "Metrics for evaluating distributed manufacturing control systems," *Computers in Industry*, vol. 51, no. 2, pp. 225–235, 2003.

[25] S. Bussmann and J. Sieverding, "Holonic control of an engine assembly plant an industrial evaluation," in *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, vol. 5, Tucson, AZ, 2001, pp. 3151–3156.

[26] J.-L. Chirn, "Developing a reconfigurable manuacturing control system – a holonic component based approach." Ph.D. dissertation, University of Cambridge, 2001.

[27] P. Leitao, "An agile and adaptive holonic architecture for manufacturing control," Ph.D. dissertation, University of Porto: Faculty of Engineering, 2004.

[28] R. Beach, A. Muhlemann, D. Price, A. Paterson, and J. Sharp, "A review of manufacturing flexibility," *European Journal of Operational Research*, vol. 122, pp. 41–57, 2000.

[29] R. Brennan, "Towards the assessment of holonic manufacturing systems," in *INCOM 2006: 12th IFAC Symposium on Information Control Problems in Manufacturing*, 2006.

[30] Chirn and D. Mcfarlane, "Proceedings of the 16th ifac world confernece," 2005.

[31] A. De Toni and S. Tonchia, "Manufacturing flexibility: a literature review," *International Journal of Production Research*, vol. 36, no. 6, pp. 1587–1617, 1998.

[32] H. A. ElMaraghy, "Flexible and reconfigurable manufacturing systems paradigms," *International Journal of Flexible Manufacturing Systems*, vol. 17, pp. 261–276, 2005.

[33] R. V. Ramasesh and M. D. Jayakumar, "Measurement of manufacturing flexibility: A value based approach," *Journal of Operations Management*, vol. 10, pp. 446–468, 1991.

[34] A. Farid, "Reconfigurability measurement in automated manufacturing systems," Ph.D. dissertation, University of Cambridge: Institute for Manufacturing, 2007.

[35] A. Farid and D. McFarlane, "A design structure matrix based method for reconfigurability measurement of distributed manufacturing systems," *International Journal of Intelligent Control and Systems*, vol. Special Issue, 2007.

[36] A. M. Farid and W. Covanich, "Measuring the Effort of a Reconfiguration Process," in *The Sixth IEEE International Conference on Industrial Informatics INDIN 2008: Harmonizing Computers, Machines and People*, Daejeon, Korea, 2008.

[37] A. M. Farid and D. C. McFarlane, "Production degrees of freedom as manufacturing system reconfiguration potential measures," *Proceedings of the Institution of Mechanical Engineers, Part B (Journal of Engineering Manufacture)*, vol. 222, no. B10, pp. 1301–1314, Oct. 2008.

[38] A. M. Farid, "Product Degrees of Freedom as Manufacturing System Reconfiguration Potential Measures," *International Transactions on Systems Science and Applications*, vol. 4, no. 3, pp. 227–242, 2008.

[39] ——, "Facilitating ease of system reconfiguration through measures of manufacturing modularity," *Proceedings of the Institution of Mechanical Engineers, Part B (Journal of Engineering Manufacture)*, vol. 222, no. B10, pp. 1275–1288, 2008.

[40] E. E. S. Baca, A. M. Farid, and I.-T. Tsai, "An Axiomatic Design Approach to Passenger Itinerary Enumeration in Reconfigurable Transportation Systems," in *Proceedings of ICAD2013 The Seventh International Conference on Axiomatic Design*, Worcester, MA, USA, 2013, p. 8.

[41] A. Viswanath, E. E. S. Baca, and A. M. Farid, "An Axiomatic Design Approach to Passenger Itinerary Enumeration in Reconfigurable Transportation Systems," *submitted to: IEEE Transactions on Intelligent Transportaton Systems*, pp. 1–10, 2013.

[42] ANSI-ISA, "Enterprise control system integration part 3: Activity models of manufacturing operations management," ISA, Research Triangle Park, North Carolina, Tech. Rep., 2005.

[43] M. Newman, *Networks: An Introduction*. Oxford University Press, Oxford UK, 2009.

[44] M. van Steen, *Graph Theory and Complex Networks: An Introduction*. Maarten van Steen, 2010, no. January.

[45] T. G. Lewis, *Network Science: Theory and Applications*. Hoboken, N.J.: Wiley, 2011.

[46] O. L. De Weck, D. Roos, and C. L. Magee, *Engineering systems : meeting human needs in a complex technological world*. Cambridge, Mass.: MIT Press, 2011.

[47] N. Suh, *Axiomatic Design: Advances and Applications*. New York: Oxford University Press, 2001.

[48] C. Cassandras and S. Lafortune, *Introduction to Discrete Event Systems*. New York: Springer, 1999.

[49] K. M. Abadir and J. R. Magnus, *Matrix algebra*. Cambridge ; New York: Cambridge University Press, 2005, no. 1.

[50] D. McFarlane, C. Carr, M. Harrison, and A. McDonald, "Auto-id's three r's: Rules and recipes for product requirements," Auto-ID Center, Cambridge, UK, Tech. Rep., 2002.