A Dynamic Model for a Cyber-Physical Healthcare Delivery System with Human Agents

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***Abstract*—The traditional monolithic hospital focused healthcare system organically developed to address acute conditions. In recent years, healthcare needs have shifted from treating acute conditions to meeting an unprecedented chronic disease burden. Chronic disease healthcare is based on continued delivery outside healthcare facilities, deep understanding of individual health state, managing individualized health needs and coordination between many medical specialities. Classic healthcare dynamic models based on production systems and applicable to acute healthcare inadequately address chronic disease. A new dynamic model paradigm is needed to capture the dynamics of providing healthcare for individuals with chronic disease based on clinical need rather than patient throughput. This paper develops a healthcare dynamic model for personalized healthcare delivery and managed individual health outcomes. It utilizes a hetero-functional graph theory rooted in Axiomatic Design for Large Flexible Engineering Systems and Petri nets. The dynamics of the model builds upon a recently developed systems architecture for healthcare delivery which bears several analogies to the architecture of mass-customized production systems. At its essence, the model consists of two synchronized Petri nets; one for the healthcare delivery system and another for individuals' health state evolution. Such a model allows for the transparent description of health outcomes and links them to the evolution of the healthcare delivery system and its associated costs.**

***Index Terms*—dynamic modeling; healthcare delivery system; individual health outcomes;**

I. Introduction

Industrial systems are often studied in terms of their structural topology. While this includes "hard" infrastructure, such as water, power, and transportation, it is important to include "soft" infrastructure [1], such as the healthcare system.

The current healthcare delivery system was designed from the outset to address acute conditions and is ill-suited to address the unprecedented chronic disease burden. Because chronic care often involves several health conditions and lasts for significantly longer periods of time, the architecture of the healthcare delivery system is likely to require change [2]. The prequel to this work [3] addressed these architectural issues; focusing on the relationship between the individual and the healthcare delivery system as well as the relationships between its many services and resources as well. This work similarly recognizes that the dynamics of a healthcare delivery system tuned to chronic conditions is also likely to require change.

With respect to healthcare system dynamics, the primary difference between the care of acute and chronic conditions is that of time scale. In acute condition care, the focus of the system is on the urgency of diagnosing and curing the physical anomalies of the patient before they fall into more serious diagnoses [4]. Because these services are spatially distributed, the transportation and inevitably the queueing of the patient within the healthcare delivery system becomes the primary concern. Consequently, a significant portion of the literature on healthcare delivery system dynamics focuses on the delays associated with these activities [5], [6]. This paradigm is particularly evident in emergency room and inpatient applications [7]–[10]. Many of these models draw on the analogous literature in production system discrete-event dynamics [11], [12]. The underlying assumption is that the operand-patient, much like the operand-product must receive transportation processes in order to be transformed by the delivery system. The health state of the individual is assumed to be quickly degrading and is assumed to apply real-time deadlines to healthcare delivery operations. In chronic conditions, however, the time associated with transportation processes is relatively short and likely negligible. Instead, it is the evolution of the individual's health state; particularly across multiple conditions that drives the need for well-coordinated care. Tracking this health state; much more than simply the individual's location is of primary importance for personalized healthcare delivery and managed individual health outcomes. These differing objectives are highlighted in the development of the dynamic system model presented in this work.

This paper develops the dynamics for a system model for personalized healthcare delivery and managed individual health outcomes. It utilizes a hetero-functional graph theory rooted in Axiomatic Design for Large Flexible Engineering Systems and Petri nets. The dynamics of the model builds upon the developed systems architecture for healthcare delivery which bears several analogies to the architecture of mass-customized production systems. At its essence, the model consists of two synchronized Petri nets; one for the healthcare delivery system and another for indviduals' health state evolution. The model applies equally to the care of both acute and chronic conditions, transparently describes health outcomes and links them to the evolution of the healthcare delivery system and its associated costs.

The development of the model rests upon the architecture foundations developed in [3]. Section II presents the essential definitions and concepts from this foundation so that Section III may develop a conceptually consistent dynamic model. Next, the Discussion is presented in Section IV and finally, the Conclusion in Section V. The work assumes prerequisite knowledge in model-based systems engineering [13]–[15], graph theory [16], and discrete-event simulation [17] which

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is otherwise gained from the cited texts.

II. Background

The development of the dynamic model in Section III rests upon the recently developed architecture for personalized healthcare delivery and managed individual health outcomes That work drew upon a hetero-functional graph theory rooted in the Axiomatic Design for Large Flexible Engineering Systems and Petri nets. The healthcare delivery system form is described by its resources in Section II-A, and its system function is described by processes in Section II-B. The processes are allocated to resources in the system concept as described by the system knowledge base in Section II-C.

A. System Form

The healthcare delivery system is composed of resources representing system form. Four types of resources $\mathbb{R} = \mathbb{R}_F \cup \mathbb{R}_D \cup \mathbb{R}_M \cup \mathbb{R}_N$ have been defined:

Definition 1. Transformation Resource: A resource $\mathbb{r}_F \in \mathbb{R}_F$ capable of a transformative effect on its operand (e.g. the health state of an individual). They include *human* transformation resources $r_F \in R_F$ (e.g. surgeon, cardiologist, psychologist) and *technical* transformation resources $\mathscr{r}_F \in \mathscr{R}_F$ (e.g. operating theaters, chemotherapy infusion room, delivery room). Transformation resources are the set union of human and technical transformation resources, $\mathbb{R}_F = R_F \cup \mathscr{R}_F$.

Definition 2. Decision Resource: A resource $\mathbb{r}_D \in \mathbb{R}_D$ capable of advising the operand, an individual, on how to proceed next with the healthcare delivery system. They include *human* decision resources $r_D \in R_D$ (e.g. oncologist, general practitioner, surgeon) and *technical* decision resources $\mathscr{r}_D \in \mathscr{R}_D$ (e.g. decision support systems, electronic medical record decision tools). Decision resources are the set union of human and technical decision resources, $\mathbb{R}_D = R_D \cup \mathscr{R}_D$.

Definition 3. Measurement Resource: A resource $\mathbb{r}_M \in \mathbb{R}_M$ capable of measuring the operand: here the health state of an individual. They include *human* measurement resources $r_M \in R_M$ (e.g. MRI technician, sonographer, phlebotomist) and *technical* measurement resources $\mathscr{r}_M \in \mathscr{R}_M$ (e.g. magnetic resonance imaging scanner, ultrasound machine, holter monitor). Measurement resources are the set union of human and technical measurement resources, $\mathbb{R}_M = R_M \cup \mathscr{R}_M$.

Definition 4. Transportation Resource: A resource $\mathbb{r}_N \in \mathbb{R}_N$ capable of transporting its operand: the individual them self. They include *human* transportation resources $r_N \in R_N$ (e.g. runners, emergency medical technician, clinical care coordinator) and *technical* transportation resources $\mathscr{r}_N \in \mathscr{R}_N$ (e.g. ambulance, gurney, wheelchair). Transportation resources are the set union of human and technical transportation resources, $\mathbb{R}_N = R_N \cup \mathscr{R}_N$.

It is useful to define the set of non-transportation related resources.

Definition 5. Buffer Resource: A resource $r \in \mathbb{R}_B$, denoting specified locations as a set union of transformation, decision and measurement resources, where

$$\mathbb{R}_B = \mathbb{R}_F \cup \mathbb{R}_D \cup \mathbb{R}_M \tag{1}$$

The healthcare delivery system resources described thus far allows specific instances to be non-uniquely classified. In the cases where a specific resource is capable of performing several processes, it is must be uniquely classified. In order to create a unique classification of these resources, a set of ordered classification rules are implemented.

Definition 6. Rules for Classification of Healthcare System Resources:

Rule 1: If $r \in R$ can *Transform*; then $r \in R_F$. If $\mathscr{r} \in \mathscr{R}$ can *Transform*; then $\mathscr{r} \in \mathscr{R}_{\mathscr{F}}$.
Rule 2: If $r \in R$ can *Decide*; then $r \in R_D$. If $\mathscr{r} \in \mathscr{R}$ can *Decide*; then $\mathscr{r} \in \mathscr{R}_{\mathscr{D}}$.
Rule 3: If $r \in R$ can *Measure*; then $r \in R_M$. If $\mathscr{r} \in \mathscr{R}$ can *Measure*; then $\mathscr{r} \in \mathscr{R}_{\mathscr{M}}$.
Rule 4: Otherwise $r \in R_N$ and $\mathscr{r} \in \mathscr{R}_{\mathscr{N}}$

B. System Function

The healthcare delivery system is composed of processes $P = P_F \cup P_D \cup P_M \cup P_N$ representing the system Function. Four types of processes have been defined [3]:

Definition 7. Transformation Process: A *physical* process $p_F \in P_F$ that transforms the operand: specifically the internal health state of the individual (i.e. treatment of condition, disease or disorder).

Definition 8. Decision Process: A *cyber-physical* process $p_D \in P_D$ occurring between a healthcare system resource and the operand: the individual, that generates a decision on how to proceed next with the healthcare delivery system.

Definition 9. Measurement Process: A *cyber-physical* process $p_M \in P_M$ that converts a physical property of the operand into a cyber, informatic property to ascertain health state of the individual.

Definition 10. Transportation Process: A *physical* process $p_N \in P_N$ that moves individuals between healthcare resources (e.g. bring individual to emergency department, move individual from operating to recovery room).

The introduction of the set of buffer resources $\mathbb{R}_B$ (in Definition 5) implies that there are $\sigma^2(\mathbb{R}_B)$ transportation processes where the $\sigma()$ notation is introduced to give the size of a set. As a matter of convention, a healthcare process p_{Nu} transports an individual from resource $r_{y_1} \in \mathbb{R}_B$ to resource $r_{y_2} \in \mathbb{R}_B$ according to the index convention [18]–[24]: $u = \sigma(\mathbb{R}_B)(y_1 - 1) + y_2$.

Definition 11. Non-Transportation Process: A combination of non-transportation processes representing transformation, decision and measurement process, $p_B \in P_B$ that is a set union of non-transportation processes. $P_B = P_F \cup P_D \cup P_M$.

C. System Concept

The system concept is defined as an allocated architecture composed of a bipartite graph between system processes and resources, that can be mathematically described as [18]–[24]. $P = J_S \odot \mathbb{R}$, where J_S is the system knowledge base.

Definition 12. System Knowledge Base [18]–[24]: A binary matrix J_S of size $\sigma(P) \times \sigma(\mathbb{R})$ whose element $J_S(w, v) \in \{0, 1\}$ is equal to one when event $e_{wv} \in \mathcal{E}_S$ (in the discrete event systems sense [17]) exists as a system process $p_w \in P$ being executed by a resource $r_v \in \mathbb{R}$. It may be calculated directly as

$$J_S = \begin{bmatrix} J_F & 0 & 0 & 0 \\ J_{FD} & J_D & 0 & 0 \\ J_{FM} & J_{DM} & J_M & 0 \\ J_{FN} & J_{DN} & J_{MN} & J_N \end{bmatrix} \tag{2}$$

The healthcare delivery system knowledge base J_S represents the elemental capabilities that *exist* within the system. These capabilities may not always be *available* and therefore such constraints can be described in a similar structure called the system events constraints matrix.

Definition 13. System Events Constraints Matrix [18]–[24]: A binary matrix K_S of size $\sigma(P) \times \sigma(\mathbb{R})$ whose element $K_S(w, v) \in \{0, 1\}$ is equal to one when a constraint eliminates event e_{wv} from the event set.

The construction of J_S and K_S allow for the construction of a *structural availability matrix* A_S describing the independent actions defining the available capabilities in the system [18]–[24]. $A_S = J_S \ominus K_S$, where $\ominus$ is Boolean subtraction. The enumeration of these independent actions defines the healthcare system's structural degrees of freedom.

Definition 14. Structural Degrees of Freedom [18]–[24]: The set of independent actions $\psi_i \in \mathcal{E}_S$ that completely defines the available processes in the system. Their number is given by:

$$DOF_S = \sigma(\mathcal{E}_S) = \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(\mathbb{R})} \left[J_S \ominus K_S\right](w, v) \tag{3}$$

From an architectural perspective, the structural degrees of freedom form the elements of a hetero-functional network [21], [25] that describes the structure of the healthcare delivery system.

It is often useful to vectorize the knowledge base, where the shorthand $()^V$ is used to replace vec(). A projection operator may be introduced to project the vectorized knowledge base onto a one's vector to eliminate sparsity. $\mathbb{P}(A_S)^V = \mathbb{1}^{\sigma(\mathcal{E}_S)}$ such that [18]–[24]:

$$\mathbb{P} = \left[e_{\psi_1}^{\sigma(\mathcal{E}_S)}, \ldots, e_{\psi_{\sigma(\mathcal{E}_S)}}^{\sigma(\mathcal{E}_S)} \right] \tag{4}$$

where $e_{\psi_1}^{\sigma(\mathcal{E}_S)}$ is the ψ_i^{th} elementary row vector corresponding to the first up to the last structural degree of freedom.

III. Dynamic Model Development

The structural model presented in the previous section provides a skeleton upon which to develop the dynamic model in this section. Because healthcare delivery systems are spatially distributed and evolve with discrete-event dynamics, the dynamic model utilizes Petri nets [17]. Two types are needed. The first is called the *Healthcare Delivery System Petri Net*. It describes the evolution of the system processes and resources of the healthcare delivery system in Section III-A. Section III-B then refines this default model to the care of chronic conditions. The second Petri net is called the *Health Net*. It describes the 'clinical' health state evolution of individuals in Section III-C. As discussed in detail previously [2], although the human body's health state evolves continuously via biological processes, the practice of clinical medicine discretizes this evolution into discrete states so as to facilitate diagnosis and decision-making. With these two Petri nets in place, their respective dynamics are synchronized in Section III-D.

A. Healthcare Delivery System Dynamics

The healthcare delivery system dynamics are described by a timed Petri net.

Definition 15. Healthcare Delivery System Petri Net: A bipartite directed graph represented as a 6-tuple: $N = \{S, E, M, W, D, Q\}$, where:

- N is the Healthcare Delivery System net.
- S is the set of places (or buffers) of size $\sigma(\mathbb{R}_B)$.
- E is the set of transitions/events of size $\sigma(\mathcal{E}_S)$.
- $M \subseteq (S \times E) \cup (E \times S)$ is the set of arcs of size $\sigma(M)$ from places to transitions and from transitions to places.
- $W : M \rightarrow \{0, 1\}$ is the weighting function on arcs.
- D is the set of transition durations.
- Q is a discrete state marking vector of size $\big(\sigma(\mathbb{R}_B) + \sigma(\mathcal{E}_S)\big) \times 1 \in \mathbb{N}^{\sigma(\mathbb{R}_B)+\sigma(\mathcal{E}_S)}$.

In the model, there is exactly one *place* for each healthcare system buffer. As many healthcare systems may have hundreds or thousands of healthcare system buffers, it is often useful to form aggregated resources $\bar{\mathbb{R}}$ [18]–[20], [23], [26].

$$\bar{\mathbb{R}} = A_R \circledast \mathbb{R} \tag{5}$$

where $\circledast$ is an aggregation operator and A_R is an aggregation matrix [18]–[20], [23], [26] and $A_R(i, j)$ iff $\mathbb{R}_j \in \bar{\mathbb{R}}_i$. For example, a human resource such as a surgeon must be aggregated with a technical resource such as an operating room in order to make a functional surgical theatre.

In the model, there is exactly one *transition* for every structural degree of freedom in the system. This allows for all the capabilities of the healthcare delivery system to be potentially engaged by the patient population. It is also important to note that the healthcare delivery system knowledge can show process redundancies where a given process can be performed by multiple resources. This critical distinction allows two different transitions to be fired and achieve the same process but engage entirely different resources at entirely different cost. For example, the process of 'perform skin suturing' performed by the resource 'resident' vs. 'plastic surgeon' have very different costs associated with each transition.

The (directed) *arcs* of the Petri net graph and its weightings define the Petri net incidence matrix M.

Definition 16. Petri Net Incidence Matrix [27]: An incidence matrix M of size $\sigma(\mathbb{R}_B) \times \sigma(\mathcal{E}_S)$: $M = M^+ - M^-$, where $M^+(y,\psi) = w(r_y, \epsilon_{wv})$ and $M^-(y,\psi) = w(r_y, \epsilon_{wv})$ and ψ is a unique index mapped from the ordered pair (w, v).

The incidence out and incidence in matrices (M^- and M^+) form the positive and negative components of the Petri net incidence matrix respectively. The incidence out matrix may be calculated straightforwardly [24].

$$\mathcal{M}^- = \sum_{y1=1}^{\sigma(\mathbb{R}_B)} e_{y1}^{\sigma(\mathbb{R}_B)} \left[\mathbb{P}\left(X_{y_1}^-\right)^V\right]^T \tag{6}$$

where

$$X_{y_1}^- = \left[\begin{array}{c|c} \mathbb{1}^{\sigma(P_B)} e_{y1}^{\sigma(\mathbb{R}_B)T} & \mathbf{0}^{\sigma(P_B)\times\sigma(\mathbb{R}_N)} \\ \hline e_{y1}^{\sigma(\mathbb{R}_B)} \quad \otimes \mathbb{1}^{\sigma(\mathbb{R}_B)} & \otimes \mathbb{1}^{\sigma(\mathbb{R})T} \end{array}\right] \tag{7}$$

Equation 6 states that the incidence matrix is the linear superposition of $\sigma(\mathbb{R}_B)$ matrices each associated with a given Petri net buffer r_{y1}. For a given buffer r_{y1}, the outer product serves to link it to its associated structural degree of freedom or equivalently a Petri net transition. Note that the matrix X_{y_1} has the same size and structure as the system knowledge base J_S and when projected by $\mathbb{P}$ (in Equation 4) serves to select out the elements aligned with the structural degrees of freedom. Finally, the X_{y_1} matrix simply places filled elements at the structural degrees of freedom that 1.) occur at r_{y1} and 2.) have r_{y1} as its origin. The incidence in matrix may be calculated analogously [24].

$$\mathcal{M}^+ = \sum_{y2=1}^{\sigma(\mathbb{R}_B)} e_{y2}^{\sigma(\mathbb{R}_B)} \left[\mathbb{P}\left(X_{y_2}^+\right)^V\right]^T \tag{8}$$

where

$$X_{y_2}^+ = \left[\begin{array}{c|c} \mathbb{1}^{\sigma(P_B)} e_{y2}^{\sigma(\mathbb{R}_B)T} & \mathbf{0}^{\sigma(P_B)\times\sigma(\mathbb{R}_N)} \\ \hline \mathbb{1}^{\sigma(\mathbb{R}_B)} \quad \otimes e_{y2}^{\sigma(\mathbb{R}_B)} & \otimes \mathbb{1}^{\sigma(\mathbb{R})T} \end{array}\right] \tag{9}$$

The Petri net structure leads directly to the definition of its timed discrete-event dynamics.

Definition 17. Timed Petri Net (Discrete-Event) Dynamics [18]–[24]: Given a binary input firing vector $U^+[k]$ and a binary output firing vector $U^-[k]$ of size both of size $\sigma(\mathcal{E}_S)\times 1$, and the positive and negative components $\mathcal{M}^+$ and $\mathcal{M}^-$ of the Petri net incidence matrix of size $\sigma(\mathbb{R}_B)\times\sigma(\mathcal{E}_S)$, the evolution of the marking vector Q is given by the state transition function $\Phi_T(Q[k], U[k])$: $Q[k+1] = \Phi_T(Q[k], U^-[k], U^+[k])$, where $Q = [Q_S; Q_{\mathcal{E}_S}]$ and

$$Q_S[k+1] = Q_S[k] \quad + \quad M^+U^+[k] \quad - \quad M^-U^-[k] \tag{10}$$

$$Q_{\mathcal{E}_S}[k+1] = Q_{\mathcal{E}_S}[k] \quad - \quad U^+[k] \quad + \quad U^-[k] \tag{11}$$

The state transition breaks the discrete state Q in two. Q_S tracks the locations of the tokens at the places $\mathbb{R}_B$ and $Q_{\mathcal{E}_S}$ tracks the locations of the tokens in the transitions $\mathcal{E}_S$ of the healthcare delivery system. The state transition function also distinguishes between input and output firing vectors so as to mark the entry and exit of tokens to and from transitions. In practice, a scheduled event list is used to implement firing vectors and ensure the durations D of each of the transitions.

Definition 18. Scheduled Event List [17]: A tuple $\mathcal{S} = (u_\psi[k], t_k)$ consisting of all elements $u_\psi[k]$ in firing vectors $U^-[k]$ and their associated times t_k. For every element, $u_\psi^-[k] \in U^-[k]$, there exists another element $u_\psi^+[\kappa] \in U^+[\kappa]$ which occurs at time t_κ, d_ψ time units later. $t_\kappa = t_k + d_\psi$.

B. The Chronic Condition Care Abstraction

The healthcare delivery system model presented in the previous subsection considered all of its inherent capabilities and integrated them within a Petri net model. Such an approach is considered sufficient for acute care. For chronic care, however, several additional considerations are required. First, because chronic conditions continue well beyond a single visit to a healthcare facility, a resource entitled 'outside clinic must be included in the model. Naturally, this will require the addition of transportation processes so as to enter and exit the clinic. Next, transportation degrees of freedom within the clinic are assumed to have a negligible duration and are therefore eliminated. K_S is modified accordingly. By Equation 3, the number of structural degrees of freedom changes as well. Consequently, a new projection operator P_C must be calculated such that:

$$\mathbb{P}_C(J_S \ominus K_S)^V = \mathbb{1}^{\sigma(\mathcal{E}_S)} \tag{12}$$

Finally, the resources within the clinic are aggregated by Equation 5 so as to yield to $\bar{\mathbb{R}}$={healthcare clinic, outside clinic}. Consequently, the healthcare delivery system Petri incidence in and incidence out matrices become:

$$\mathcal{M}^- = A_R \sum_{y1=1}^{\sigma(\mathbb{R}_B)} e_{y1}^{\sigma(\mathbb{R}_B)} \left[\mathbb{P}_{\mathbb{C}}\left(X_{y_1}^-\right)^V\right]^T \tag{13}$$

$$\mathcal{M}^+ = A_R \sum_{y2=1}^{\sigma(\mathbb{R}_B)} e_{y2}^{\sigma(\mathbb{R}_B)} \left[\mathbb{P}_{\mathbb{C}}\left(X_{y_2}^+\right)^V\right]^T \tag{14}$$

This hierarchical aggregation implements the chronic condition care abstraction. The focus now becomes the various forms of transformation, decision, and measurement processes that the patient receives rather than transportation and queuing within the clinic.

C. Health Net Dynamics

As mentioned previously, the Health Net is introduced so as to represent the clinical health state of individuals.

Definition 19. Health Net: Given an individual l_i, that is part of a population L, where $L = \{l_1, ..., l_{\sigma(L)}\}$, the evolution of their clinical health state can be described as a fuzzy timed Petri net [28]–[30]: $N_{l_i} = \{S_{l_i}, E_{l_i}, M_{l_i}, W_{l_i}, D_{l_i}, Q_{l_i}\}$, where:

- N_{l_i} is the health net.
- S_{l_i} is the set of places describing a set of health states.
- E_{l_i} is the set of transitions describing health events.
- $M_{l_i} \subseteq (S_{l_i} \times E_{l_i}) \cup (E_{l_i} \times S_{l_i})$ is the set of arcs describing the relations of (health states to health events) or (health events to health states).
- W_{l_i} is the set of weights on the arcs describing the health transition probabilities for the arcs.
- D_{l_i} is the set of transition durations.
- Q_{l_i} is the Petri net marking representing the likely presence of the set of health states as a discrete probabilistic state.

The Petri net structure leads directly to the definition of its discrete-event dynamics.

Definition 20. Fuzzy Timed Petri Net (Discrete-Event) Dynamics [31], [32]: Given a binary input firing vector $U^+[k]$ and a binary output firing vector $U^-[k]$ both of size $\sigma(\mathcal{E}_{l_i})\times 1$, and the positive and negative components $M^+_{l_i}$ and $M^-_{l_i}$ of the Petri net incidence matrix of size $\sigma(S_{l_i})\times\sigma(\mathcal{E}_{l_i})$, the evolution of the marking vector Q_{l_i} is given by the state transition function $\Phi(Q_{l_i}[k], U[k])$: $Q_{l_i}[k+1] = \Phi(Q_{l_i}[k], U^-[k], U^+[k])$, where $Q_{l_i} = [Q_{Sl_i}; Q_{\mathcal{E}l_i}]$ and

$$Q_{Sl_i}[k+1] = Q_{Sl_i}[k] \quad + \quad M^+U^+[k] \quad - \quad M^-U^-[k] \tag{15}$$

$$Q_{\mathcal{E}l_i}[k+1] = Q_{\mathcal{E}l_i}[k] \quad - \quad U^+[k] \quad + \quad U^-[k] \tag{16}$$

Q_{Sl_i} is introduced to probabilistically mark Petri net places where as $Q_{\mathcal{E}l_i}$ is introduced to mark the likelihood that a timed transition is currently firing. The transitions are fired based on a scheduled event list that combines the discrete events with a time interval as described in Definition 18.

D. Coordination of the Healthcare Delivery System Petri Net & Individual Health Net Dynamics

As expected, the healthcare delivery system Petri net and the health net dynamics are inherently coupled. Each transformation process in the healthcare delivery system induces its corresponding health event. For each individual, l_i, this feasibility condition can be captured in a binary individual transformation feasibility matrix [3] .

Definition 21. Individual Transformation Feasibility Matrix Λ_{F_i} [18]–[24], [32]: a binary matrix of size $\sigma(E_{l_i})\times\sigma(P_F)$, where $\Lambda_{F_i}(x, j) = 1$ if transformational process p_{F_j} realizes the health event e_{xl_i}.

An individual firing matrix is introduced to synchronize the healthcare delivery system Petri net firing vectors with those of the (individual) health nets.

Definition 22. Individual Health Firing Matrix [24], [33]: A binary individual health firing matrix $\mathcal{U}[k]$ of size $\sigma(\mathcal{E}_S)\times\sigma(L)$, whose element $u_{\psi,l}[k] = 1$ when the k^{th} firing timing triggers an individual l to take structural degree of freedom ψ for action.

Consequently, the healthcare delivery system input firing vectors at a given moment k become $U^- = \mathcal{U}\mathbb{1}^{\sigma(L)}$ and each health net firing vector at a given moment k becomes $\Lambda^T_{F_i} \cdot U_{l_i} = \mathcal{A}_F \cdot \mathcal{U} \cdot e^{\sigma(L)T}_{l_i}$ and $\mathcal{A}_F$ serves to select out the structural degrees of freedom associated with transformation.

IV. Results & Discussion

The dynamic model for personalized healthcare delivery and managed health outcomes has several important aspects: 1.) it applies equally to the care of both acute and chronic conditions; 2.) it transparently describes health outcomes; and 3.) it transparently links cost and outcomes.

First, the model applies to both acute and chronic care. The acute care dynamic modeling resembles those commonly found in industrial engineering and operations research literature [5], [6]. It emphasizes the importance of scheduling and minimized queuing. At the timescale of a clinic visit, acute care decision processes like care planning and scheduling are critical. The timeliness of decision-making was highlighted in the acute care the orthopedic case. Acute care requires a more granular resolution of healthcare delivery system capabilities (i.e. structural degrees of freedom). Consequently, many more are utilized per visit relative to chronic care. The utilization of spatially distributed transformative, decision, and measurement capabilities within a short period of time naturally raises questions of transportation (e.g. in emergency rooms) and queues (e.g. in patient care). In chronic care, these concerns are diminished. The model abstracts away transportation so as to focus on the coordination of transformation, decision and measurement processes.

The health net in this model is an important contribution that serves to transparently describe health outcomes. In acute care, the health net tends to cycle back to an initially healthy state in a fairly short period of time; and perhaps within a single visit. In chronic care, not only are multiple clinical visits required but the state of the individual's health must be tracked in the meantime. While in some chronic conditions a return to a healthy state is possible, in most instances the healthcare delivery system must actively track and manage its degradation.

Indeed, the most important aspect of the model is its coherence between the healthcare delivery system and the individual's health state. The states of the Petri nets are tied directly and should ideally be coordinated in order to deliver effective care. Whether for acute or chronic conditions, time is of the essence. Because the health nets have stochastic processes that will fire spontaneously, the healthcare delivery system must take timely and coordinated action to avoid adverse and negative health outcomes.

Finally, it is important to recognize that each healthcare delivery system degree of freedom incurs a cost every time it is fired. Therefore, as the two Petri nets evolve simultaneously, the discrete event simulation transparently reveals the accumulation of incurred cost versus the evolution of health outcomes.

V. Conclusion & Future Work

In conclusion, this paper develops the dynamic system model for personalized healthcare delivery and managed individual health outcomes. The dynamics of the model rests upon the developed systems architecture from prior work. This work draws upon a hetero-functional graph theory rooted in Axiomatic Design for Large Flexible Engineering Systems and Petri nets. The dynamic model coordinates the healthcare delivery system and the individual net. The healthcare delivery net evolves as the transitions fire when the system is utilized, while the individual net evolves as the individual's health state evolves due to the spontaneous firing of stochastic process and as the individual receives transformative processes by the healthcare delivery system.

The development of the model opens several new avenues for future work. The Petri net firing vectors indicated as inputs to the model provide an opportunity for the development of rigorous decision-making algorithms. The clear trade-offs between cost and health outcomes is likely to be interest to many healthcare delivery system stakeholders including clinicians, healthcare facility administrators, insurance companies, and regulators. Finally, as the need for such a model matures, new approaches to automated model building that integrate with healthcare enterprise information systems is likely to grow.

References

[1] W. A. Niskanen, "The Soft Infrastructure of a Market Economy," *Cato Journal*, vol. 11, no. 2, pp. 233–238, 1991.

[2] I. S. Khayal and A. M. Farid, "The Need for Systems Tools in the Practice of Clinical Medicine," *Systems Engineering*, 2017. [Online]. Available: http://dx.doi.org/10.1002/sys.21374

[3] I. Khayal and A. Farid, "An Architecture for a Cyber-Physcal Healthcare Delivery System with Human Agents," in *Proceedings of the 2017 IEEE International Conference on Systems, Man, and Cybernetics (SMC2017), Intelligent Industrial System Special Issue*, October 2017.

[4] S. F. I. f. H. &. A. University of California and R. W. J. Foundation, *Chronic Care in America: A 21st Century Challenge*. Robert Wood Johnson Foundation, 1996. [Online]. Available: https://books.google.com/books?id=Ek1rAAAAMAAJ

[5] M. M. Günal and M. Pidd, "Discrete event simulation for performance modelling in health care: a review of the literature," *Journal of Simulation*, vol. 4, no. 1, pp. 42–51, 2010.

[6] S. H. Jacobson, S. N. Hall, and J. R. Swisher, "Discrete-event simulation of health care systems," in *Patient flow: Reducing delay in healthcare delivery*. Springer, 2006, pp. 211–252.

[7] Y. N. Marmor, B. Golany, S. Israelit, and A. Mandelbaum, "Designing patient flow in emergency departments," *IIE Transactions on Healthcare Systems Engineering*, vol. 2, no. 4, pp. 233–247, oct 2012. [Online]. Available: http://dx.doi.org/10.1080/19488300.2012.736118

[8] J. S. Peck, J. C. Benneyan, D. J. Nightingale, and S. A. Gaehde, "Characterizing the value of predictive analytics in facilitating hospital patient flow," *IIE Transactions on Healthcare Systems Engineering*, vol. 4, no. 3, pp. 135–143, jul 2014. [Online]. Available: http://dx.doi.org/10.1080/19488300.2014.930765

[9] M. Kaner, T. Gadrich, S. Dror, and Y. N. Marmor, "Generating and evaluating simulation scenarios to improve emergency department operations," *IIE Transactions on Healthcare Systems Engineering*, vol. 4, no. 3, pp. 156–166, jul 2014. [Online]. Available: http://dx.doi.org/10.1080/19488300.2014.938281

[10] a. J. Rivera and B.-T. Karsh, "Human factors and systems engineering approach to patient safety for radiotherapy." *International journal of radiation oncology, biology, physics*, vol. 71, no. 1 Suppl, pp. S174–S177, 2008.

[11] C. G. Cassandras and S. Lafortune, *Introduction to Discrete Event Systems*, 2nd ed. New York, NY, USA: Springer, 2007.

[12] J. Banks, *Discrete-event system simulation*, 4th ed., J. Banks, Ed. Upper Saddle River, N.J.: Pearson Prentice Hall, 2005.

[13] D. M. Buede, *The engineering design of systems: models and methods*. Hoboken, NJ: John Wiley & Sons, 2011, vol. 55.

[14] T. Weilkiens, *Systems engineering with SysML/UML: modeling, analysis, design*. Burlington, MA: Morgan Kaufmann, 2011.

[15] S. Friedenthal, A. Moore, and R. Steiner, *A practical guide to SysML: the systems modeling language*. Morgan Kaufmann, 2014.

[16] M. Van Steen, *Graph theory and complex networks: An Introduction*. Maarten van Steen, 2010, vol. 144.

[17] C. G. Cassandras, *Introduction to discrete event systems*. Springer Science & Business Media, 2008.

[18] A. M. Farid, "Reconfigurability measurement in automated manufacturing systems," Ph.D. dissertation, University of Cambridge, 2007.

[19] A. M. Farid and D. C. McFarlane, "Production degrees of freedom as manufacturing system reconfiguration potential measures," *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, vol. 222, no. 10, pp. 1301–1314, 2008.

[20] A. M. Farid, "Product Degrees of Freedom as Manufacturing System Reconfiguration Potential Measures," *International Transactions on Systems Science and Applications – invited paper*, vol. 4, no. 3, pp. 227–242, 2008.

[21] ——, "Static Resilience of Large Flexible Engineering Systems : Axiomatic Design Model and Measures," *IEEE Systems Journal (in press)*, vol. PP, no. 99, pp. 1–12, 2015. [Online]. Available: http://amfarid.scripts.mit.edu/resources/Journals/IES-J19.pdf

[22] ——, "An Axiomatic Design Approach to Non-Assembled Production Path Enumeration in Reconfigurable Manufacturing Systems," in *2013 IEEE International Conference on Systems Man and Cybernetics*, Manchester, UK, 2013, pp. 1–8. [Online]. Available: http://dx.doi.org/10.1109/SMC.2013.659

[23] A. M. Farid and L. Ribeiro, "An Axiomatic Design of a Multi-Agent Reconfigurable Mechatronic System Architecture," *IEEE Transactions on Industrial Informatics*, vol. 11, no. 5, pp. 1142–1155, 2015. [Online]. Available: http://dx.doi.org/10.1109/TII.2015.2470528

[24] W. C. Schoonenberg and A. M. Farid, "A dynamic production model for industrial systems energy management," in *2015 IEEE International Conference on Systems, Man, and Cybernetics (SMC),*. IEEE, 2015, pp. 1–7.

[25] A. M. Farid, "An Engineering Systems Introduction to Axiomatic Design," in *Axiomatic Design in Large Systems: Complex Products, Buildings & Manufacturing Systems*, A. M. Farid and N. P. Suh, Eds. Berlin, Heidelberg: Springer, 2016, ch. 1, pp. 1–47. [Online]. Available: http://dx.doi.org/10.1007/978-3-319-32388-6

[26] I. S. Khayal and A. M. Farid, "Axiomatic Design Based Volatility Assessment of the Abu Dhabi Healthcare Labor Market," *Journal of Enterprise Transformation*, vol. 5, no. 3, pp. 162–191, 2015. [Online]. Available: http://dx.doi.org/10.1080/19488289.2015.1056449

[27] A. M. Farid, "Multi-agent system design principles for resilient coordination & control of future power systems," *Intelligent Industrial Systems*, vol. 1, no. 3, pp. 255–269, 2015.

[28] W. Pedrycz and H. Camargo, "Fuzzy timed Petri nets," *Fuzzy Sets and Systems*, vol. 140, no. 2, pp. 301–330, 2003. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0165011402005249

[29] Z. Ding, H. Bunke, M. Schneider, and A. Kandel, "Fuzzy timed petri net definitions, properties, and applications," *Mathematical and Computer Modelling*, vol. 41, no. 2–3, pp. 345–360, 2005. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0895717705000853

[30] Z. Ding, H. Bunke, O. Kipersztok, M. Schneider, and A. Kandel, "Fuzzy timed Petri nets — analysis and implementation," *Mathematical and Computer Modelling*, vol. 43, no. 3–4, pp. 385–400, 2006. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0895717705005546

[31] L. Popova-Zeugmann, *Time Petri Nets*. Berlin, Heidelberg: Springer, 2013.

[32] I. Khayal and A. Farid, "Architecting a System Model for Personalized Healthcare Delivery and Managed Individual Health Outcomes," *Submitted to Complexity*, 2017.

[33] A. M. Farid, "A Hybrid Dynamic System Model for Multi-Modal Transportation Electrification," *IEEE Transactions on Control System Technology (in press)*, vol. 1, no. 1, pp. 1–12, 2016. [Online]. Available: http://dx.doi.org/10.1109/TCST.2016.2579602