

# The Impact of Wind Power Geographical Smoothing on Operating Reserve Requirements

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**Abstract**—As one of the common sources of renewable energy generation worldwide, wind power is the subject of numerous academic and industrial studies. The impact of the wind generation on different aspects of power system operations is extensively addressed in the literature. One of the important aspects of such studies is related to the dynamics of the wind generation that emerges when the wind turbines are arranged into arrays. Arranging wind turbines into arrays alters the output wind generation due to the emerging coupling between wind turbines and the geographical smoothing. While the impact of the coupling between wind turbines on the output magnitudes is well studied, the impact of geographical distribution of turbines on wind power variability has received little attention. Moreover, the impact of the geographical smoothing on the operating reserve requirements is omitted. The studies show that the variability and the forecast error of the wind power are two main factors defining the operating reserve requirements. This paper studies the impact of the geographical distribution of wind turbines on variability and the requirements of three types of operating reserves, namely, load following, ramping and regulation. The results show that the geographical distribution narrows the power spectrum of the turbine output and, therefore, reduces the variability. As a result, all three types of operating reserve requirements are reduced significantly.

## I. INTRODUCTION

As one of the major sources for renewable energy generation worldwide, wind power is the subject of numerous academic and industrial studies. The impact of the wind generation on different aspects of power system operations is extensively addressed in the literature [1]–[5]. One of the important aspects of such studies is related to the dynamics of the wind generation that emerges when the wind turbines are arranged into arrays. Arranging wind turbines into arrays alters the output wind generation due to two factors. First, geographical proximity of turbines creates coupling between them that affects the magnitudes of the individual turbine outputs [6]. Second, the geographical distribution of the generation sources weakens the correlation between the outputs that results in smoother profile power output.

The impact of wind turbine spacing on magnitudes of individual turbine outputs is addressed in the literature [7]–[9]. These studies are focused on determining the output

magnitudes of wind turbines depending on their position in the array. One study [6] addresses the effect of the stream-wise and span-wise spacings in the fully developed regime, i.e., for wind-farms that are sufficiently extended so that the turbine power output becomes independent of the downstream position. Arranged and staggered turbine configurations are studied. The results show that for an aligned wind-farm, the stream-wise spacing between the turbines is the most important parameter for determining power output. However, for a staggered configuration, the geometric mean turbine spacing has the dominant impact on power output. Moreover, the output magnitudes of individual wind turbines is independent on its downstream position passing a certain point. These findings are essential for proper capacity planning of the power system since they allow estimation the capacity factor of the installed wind generation.

While the impact of the coupling between wind turbines on the output magnitudes is well studied, the impact of the geographical distribution of wind turbines on wind power variability has received little attention. Moreover, the impact of the geographical smoothing on the operating reserve requirements is omitted. Previous studies show that the variability and the forecast error of the wind power are two main factors defining the operating reserve requirements [10]–[12]. This paper studies the impact of the geographical distribution of wind turbines on variability and the requirements of three types of operating reserves, namely, load following, ramping and regulation. The major part of derivations are performed in the spectral domain. The results show that the geographical distribution narrows the power spectrum of the wind power output and, therefore, reduces the variability. As a result, all three types of operating reserve requirements are reduced significantly.

This paper is organised as follows. Section II presents the concepts and definitions necessary for the development of the methodology. Section III describes the methodology used to obtain the variability and the operating reserve requirements of the distributed wind generation. Section IV describes the case study and the scenarios studied in this paper. Section V presents the obtained results and their discussion. Section VI concludes the paper and describes the future work.

## II. BACKGROUND

The geographical distribution of wind power weakens the correlation between individual turbine outputs, which leads to a smoother profile of cumulative power output [13]. Smoother

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profile guarantees less variability of the wind generation. This paper uses the following definition of variability [4], [5]:

**Definition 1.** Variability ( $A$ ): Given the choice of the output  $P(t)$  (e.g. the wind generation, the load, the net load), the variability is the root-mean-square of that output's rate normalized by the root-mean-square of that output:

$$A = \frac{\text{rms}(dP(t)/dt)}{\text{rms}(P(t))} \quad (1)$$

Equation (1) shows that the variability describes the rate of change of the profile and has a unit of measurement of  $\text{min}^{-1}$ . The definition of variability is often presented in the frequency domain that reveals some important aspects following from (1). Also, it is known from the literature that wind and load power spectra have distinctive shapes [14], [15], which makes a frequency domain representation of variability more flexible. Thus, using the Parseval's theorem, the definition of the variability in the frequency domain takes the following form:

$$A = \sqrt{\frac{\int_{-\infty}^{\infty} \omega^2 G(\omega) d\omega}{\int_{-\infty}^{\infty} G(\omega) d\omega}} \quad (2)$$

where  $G(\omega)$  is the power spectrum of  $P(t)$  profile. Equation (2) indicates that the variability corresponds to the width of the power spectrum.

Smoothing of the wind generation profile also affects the operating reserve requirements. Previous studies show that the variability and the forecast error are two main factors defining the reserve requirements [10]–[12]. Three types of reserves are studied in this paper, namely, load following, ramping and regulation. This paper uses the classification of reserves found in [16], [17]. The reserves are defined as follows [4], [5]:

**Definition 2.** Load following reserve: *Capacity available during normal operations for assistance in active power balance to correct the future anticipated imbalance (upward and downward).*

**Definition 3.** Ramping reserve: *Capacity available for assistance in active power balance during infrequent events that are more severe than balancing needed during normal conditions and is used to correct non-instantaneous imbalances (upward and downward).*

**Definition 4.** Regulation reserve: *Capacity available during normal operations for assistance in active power balance to correct the current imbalance (upward and downward).*

It is important to add that the load following and ramping reserves operate in the day-ahead scheduling time scale  $T_h$ , while the regulation operates in the real-time balancing time step  $T_m$  [4], [5].

### III. METHODOLOGY

The cumulative power output from  $N$  wind turbines can be written as:

$$\tilde{P}(t) = \sum_{n=1}^N P_n(t) \quad (3)$$

Geographical proximity makes the individual turbine outputs  $P_n(t)$  related to each other. However, the time domain representation in (3) leaves little flexibility for utilizing these relations. To that end, the derivations in this section are conducted in the frequency domain. It is known that the power spectra of wind power and load have distinctive shapes [14], [15]. Here, the truncated Fourier transform of a stand-alone wind turbine with a unit capacity is denoted as  $P_0(\omega)$ . Also, the coupling of wind turbine output magnitudes is also incorporated into the derivations. The parameter  $\gamma_n$  indicates the reduction of the magnitude of the turbine  $n$  in the array compared to the stand-alone case. Thus, if the turbines in the array have capacities  $C_n$ , the output of each turbine can be written as:

$$P_n(\omega) = C_n \gamma_n P_0(\omega) e^{i\phi_n(\omega)} \quad (4)$$

where the phases  $\phi_n(\omega)$  occur due to the spatial distribution of the turbines. Substituting (4) into (3), the cumulative power output in the frequency domain takes the following form:

$$\tilde{P}(\omega) = \sum_{n=1}^N C_n \gamma_n P_0(\omega) e^{i\phi_n(\omega)} = P_0(\omega) \sum_{n=1}^N C_n \gamma_n e^{i\phi_n(\omega)} \quad (5)$$

In the absence of spatial distribution, all turbines are virtually placed on the same spot. As a result, the phase difference disappear  $\phi_n(\omega) = 0$ . Also, since the coupling between the turbine outputs vanishes, the magnitudes remain unchanged  $\gamma_n = 1$ . Thus, in the absence of spatial distribution, (5) takes the following form:

$$P(\omega) = P_0(\omega) \sum_{n=1}^N C_n = C \cdot P_0(\omega) \quad (6)$$

where

$$C = \sum_{n=1}^N C_n \quad (7)$$

is the total installed capacity. To establish a relation between the outputs with (5) and without (6) spatial distribution, normalized capacities are defined as follows:

$$c_n = \frac{C_n}{C} = \frac{C_n}{C_1 + C_2 + \dots + C_N} \quad (8)$$

Using (8), (5) can be written as:

$$\begin{aligned} \tilde{P}(\omega) &= P_0(\omega) \sum_{n=1}^N C_n \gamma_n e^{i\phi_n(\omega)} = C P_0(\omega) \sum_{n=1}^N c_n \gamma_n e^{i\phi_n(\omega)} = \\ &= g(\omega) \cdot P(\omega) \end{aligned} \quad (9)$$

where

$$g(\omega) = \sum_{n=1}^N c_n \gamma_n e^{i\phi_n(\omega)} \quad (10)$$

is the *smoothing function* and is at the center of this study. Using (5), the power spectrum of the output can be written as:

$$\begin{aligned}\tilde{G}(\omega) &= \tilde{P}(\omega) \cdot \tilde{P}^*(\omega) = g(\omega)P(\omega)g^*(\omega)P^*(\omega) = \\ &= |g(\omega)|^2 G(\omega)\end{aligned}\quad (11)$$

According to (2), the variability takes the following form:

$$A = \sqrt{\frac{\int_{-\infty}^{\infty} \omega^2 |g(\omega)|^2 G(\omega) d\omega}{\int_{-\infty}^{\infty} |g(\omega)|^2 G(\omega) d\omega}}\quad (12)$$

Thus, multiplication by the smoothing function alters the spectral width and, hence, the variability.

Finally, explicit representations for  $\phi_n(\omega)$  phases should be chosen to complete the model. This study uses a simple approach that assumes the wind hitting each turbine is delayed by time  $\tau_n$  due to the spatial distribution along a line. Thus, the phases can be modeled as:

$$\phi_n(\omega) = \omega \cdot \tau_n \quad (13)$$

and the smoothing function takes the following explicit form:

$$g(\omega) = \sum_{n=1}^N c_n \gamma_n e^{i\omega \tau_n} \quad (14)$$

Thus, the shape of the smoothing function depends on the temporal parameters  $\tau_n$ , and the capacities  $c_n$  and  $\gamma_n$ . Since the smoothing function appears in both the numerator and the denominator of the variability (12), its magnitude is of a small interest and the normalized smoothing function will be studied here.

#### IV. CASE STUDY

The purpose of this paper is to study the impact of geographical smoothing on the operating reserve requirements. To achieve that, the characteristics of the smoothing function are studied for different temporal parameters and capacities. Next, the smoothing function with favorable parameters is used to study the impact of geographical smoothing on the requirements of three types of operating reserves, namely, load following, ramping and regulation. Thus, the following three cases are studied:

- *The impact of  $\tau_n$  temporal parameters on the smoothing function and the variability*
- *The impact of  $c_n$  normalized capacities on the smoothing function and the variability*
- *The impact of geographical smoothing on operating reserve requirements*

Since  $\gamma_n$  parameters appear as multipliers to  $c_n$  capacities, they are of a little interest for this study and are assumed as  $\gamma_n = 1$ . All scenarios in this case study have  $N = 100$  turbines.

##### A. The Impact of Temporal Parameters

For this case, all turbines are taken to have the same capacity. Thus, from (8):

$$c_n = 1/N, \quad n = 1, \dots, N \quad (15)$$

Two aspects of the temporal distribution of wind turbines are of the most interest here, namely, the impact of the width and the uniformity of the temporal distribution on the smoothing function and the variability. To that end, the following three scenarios are studied:

- **Scenario A1.** The wind turbines are uniformly distributed within the given  $T$  interval. Thus,  $\tau_n$  can be written as:

$$\tau_n = T \cdot n/N; \quad n = 1, \dots, N \quad (16)$$

$T = 2000s$  is taken for the first scenario.

- **Scenario A2.** The second scenario uses the same uniform distribution in (16) with different width  $T = 1000s$ .
- **Scenario A3.** The goal of the third scenario is to compare the results for the uniform distribution in (16) with the case where wind turbines are clustered. To that end, the turbines are as clustered into two tight groups within  $T = 2000s$  interval as follows:

$$\tau_n = \begin{cases} T \cdot \frac{n}{2N} & \text{if } n \leq N/2, \\ T \cdot \frac{n+N}{2N} & \text{if } N/2 < n \leq N. \end{cases} \quad (17)$$

It should be noted, that while the turbines are clustered into two tight groups, they still cover  $T = 2000s$  interval.

The results of these three scenarios reveal which temporal distribution maximizes the smoothing effect.

##### B. The Impact of Normalized Capacities

For this case, wind turbines are taken to have uniform temporal distribution as in (16) with  $T = 2000s$ . Similar to the previous case, two aspects are studied here. First, the comparison of systems with identical and differing capacities. Second, in the case of differing capacities, what distribution of the turbines with different capacities results in better smoothing. To that end, the following three scenarios are studied:

- **Scenario B1.** All wind turbines have the same capacity as in (15).
- **Scenario B2.** The purpose of the second scenario is to study the impact of turbines with different capacities. To this end, the wind turbines are split into two equal groups, where  $N/2$  turbines in the first group have  $C_n = C_0$  capacities, while other  $N/2$  turbines in the second group have  $C_n = KC_0$  capacities. Thus, according to 7, the installed capacity is:

$$C = N/2 \cdot C_0 + N/2 \cdot KC_0 = C_0 \cdot (K+1)N/2 \quad (18)$$

Using (8), the normalized capacities take the following form:

$$c_n = \begin{cases} \frac{2}{(K+1)N} & \text{if } n \leq N/2, \\ \frac{2K}{(K+1)N} & \text{if } N/2 < n \leq N. \end{cases} \quad (19)$$

$K = 50$  is taken for this scenario.

- **Scenario B3.** This scenario is very similar to the previous one with one crucial difference. Here also the turbines are grouped into two groups with different capacities. The important difference is that these two groups are not clustered together in the same area. Both groups are uniformly distributed in the given  $T = 2000s$  interval. Thus,  $c_n$  have the following form:

$$c_n = \begin{cases} \frac{2}{(K+1)N} & \text{for random } N/2 \text{ turbines,} \\ \frac{2K}{(K+1)N} & \text{for other } N/2 \text{ turbines.} \end{cases} \quad (20)$$

The difference from (19) is that both group of turbines are uniformly distributed in the given area.

The results of these three scenarios reveal what ratio of capacities maximizes the smoothing effect.

### C. The Impact of Geographical Smoothing on Operating Reserve Requirements

When the favorable parameters are obtained for the smoothing function (10), the paper proceeds to the main interest of this study, namely, the impact of the geographical smoothing on the operating reserve requirements. Load following, ramping and regulation reserves are considered in this paper. The studies show, that the requirements of all these three type of reserves depend on the variability [10]–[12] and their amounts decrease as the variability decreases. To this end, geographical smoothing can have substantial economic benefits from reduced expenses on the reserve procurement.

For this case, all turbines are taken to have the same capacity as in (15) and the uniform temporal distribution as in (16). The value of  $T$  is changed for a wide range of values and for each value the variability of the wind generation and the requirements of three types of reserves are calculated. The calculations of reserves are done analytically using methodologies proposed in [10]–[12]. The load and wind data are taken from Bonneville Power Administration (BPA) repositories [18]. For this case, the day-ahead scheduling time step is taken  $T_h = 60min$  and the real-time balancing time step  $T_m = 5min$ .

## V. THE RESULTS

This section presents the results in accordance to the case study described in the previous section.

### A. The Impact of Temporal Parameters

As described in Section IV-A, three scenarios are simulated to study the impact of the temporal parameters on the smoothing function and the variability. The results are presented in Fig. 1. The horizontal axis here represents the actual frequency  $f = \omega/2\pi$  measured in  $Hz$  instead of  $\omega$  to demonstrate the mapping between the value of parameter  $T$  and the characteristic frequencies of the smoothing function.

The comparison of the graphs for the first two scenarios shows that the width of the smoothing function for the first

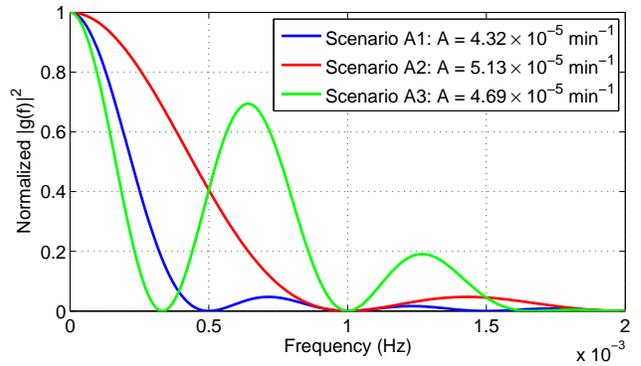


Fig. 1. The impact of temporal parameters on the smoothing function and the variability

scenario is narrower than that for the second scenario. To be more precise, the graph for the second scenario replicates the graph for the first scenario, but stretched twice in the horizontal direction. This is due to the fact, that the value of  $T$  is twice as big for the first scenario as for the second scenario. Moreover, an observation of Fig. 1 shows that the frequencies where the smoothing function reaches first zero value matches the inverse of  $T$ . For the first scenario,  $f_0 = 1/2000s = 0.5 \times 10^{-3} Hz$ , and for the second scenario  $f_0 = 1/1000s = 10^{-3} Hz$ . Thus, it can be concluded that the larger value of  $T$  corresponds to narrower smoothing function.

Also, Fig. 1 shows that the main lobe is narrower for the third scenario. However, the side lobes now are significantly higher than for the first two scenarios. Thus, it can be concluded that clustering of turbines results in high side lobes. These results stay consistent for different numbers of wind turbines  $N$ .

According to (2), the variability corresponds to the width of the power spectrum. According to (11), the power spectrum of the wind farm is multiplied by the smoothing function. Therefore, for a better smoothing and reduction of variability, the desired smoothing profile should have narrow main lobe and low side lobes. Comparing the results for variabilities in Fig. 1 shows that the variability for the first scenario is lower than the variability for the second scenario since it has narrower main lobe, while the side lobes are at the same levels. The results for the third scenario are affected by both the width of the main lobe and the height of the side lobes. As a result, while it has the narrowest main lobe, its variability is still biggest due to high side lobes. Thus, it can be concluded that the best smoothing is achieved for the uniform temporal distribution and high values of  $T$ .

### B. The Impact of Normalized Capacities

As described in Section IV-B, three scenarios are simulated to study the impact of the normalized capacities on the smoothing function and the variability. The results are presented in Fig. 2. Similar to the previous case, the horizontal axis represents the actual frequency  $f$ .

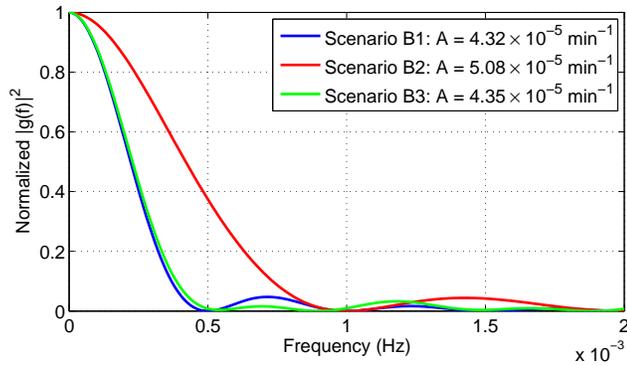


Fig. 2. The impact of normalized capacities on the smoothing function and the variability

The comparison of the graphs for the first two scenarios shows that the width of the smoothing function for the first scenario is narrower than that for the second scenario. Moreover, the graph for the second scenario seems to replicate the graph for the first scenario, but stretched twice in the horizontal direction. A similar situation was also observed in the previous case. However, in the previous case, the temporal parameter  $T$  was twice different for considered scenarios, which is not the case here. While the value of  $T = 2000s$  for both scenarios, the behavior of the graph for the second scenario correlates more with  $T = 1000s$ . This phenomenon is explained by the fact, that while the wind farm occupies the  $T = 2000s$  interval, half of that is occupied by turbines that have significantly smaller capacity than the other half of turbines. As a result, the turbines that have substantial contribution only occupy the  $T = 1000s$  interval, which is reflected in the Fig. 2 results.

Also, Fig. 2 shows that the graph for the third scenario nearly matches the graph for the first scenario. It is important to notice, that the difference between the second and third scenarios is that, for the third scenario, the wind turbines of different capacities are distributed uniformly in the given area instead of being clustered into groups. These results also stay consistent for different numbers of wind turbines  $N$ .

The variability results in Fig. 2 also confirm these conclusions. The variabilities for the first and the third scenarios are matching closely, while the variability for the second scenario is noticeably higher. Thus, it can be concluded, that the best smoothing occurs when either all turbines have the same capacity, or, in the case of different capacities, they are distributed uniformly in the given area.

### C. The Impact of Geographical Smoothing on Operating Reserve Requirements

As described in Section IV-C, this section studies the impact of geographical smoothing on load following, ramping and regulation reserve requirements. The results for the regulation reserve requirement are presented in Fig. 3. The graph shows that the regulation reserve requirements decrease as the parameter  $T$  increases. However, at some point the graph goes to saturation and leaving the regulation reserve requirement

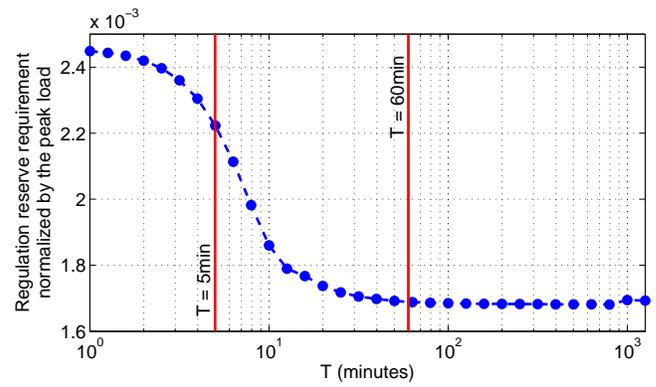


Fig. 3. The impact of geographical smoothing on regulation reserve requirement

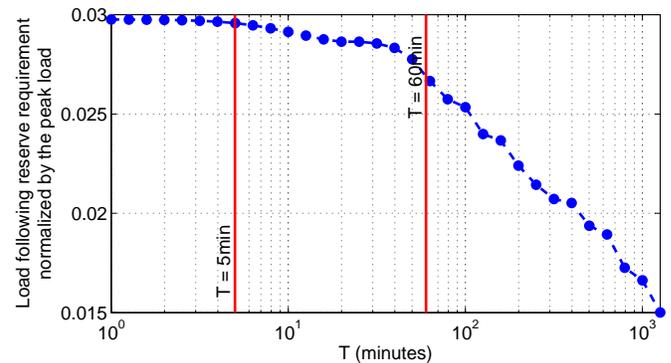


Fig. 4. The impact of geographical smoothing on load following reserve requirement

unchanged. The graph doesn't reach zero, since besides the wind generation, the regulation reserve is required to mitigate the imbalances caused by the system load. The saturation level corresponds to the regulation reserve required by the system with no wind generation.

An interesting fact about Fig. 3 is that the major decline in the regulation reserve requirement happens around  $T = 5min$  which matches the real-time balancing time step  $T_m$  described in Section IV-C. As studied above, the value of  $T$  defines the width of the smoothing function. The smoothing function is multiplied by the power spectrum and narrows it by filtering out all dynamics faster than the characteristic time  $T$ . When  $T > T_m$ , the effective power profile lacks dynamics faster than the balancing time step  $T_m$ . As a result, the requirement for the regulation reserve drops significantly, since its role is to mitigate dynamics at time scales faster than  $T_m$ .

Next, the impact of geographical smoothing on the load following reserve requirement is studied. The results are presented in Fig. 4. Similar to the previous case, the load following reserve requirement decreases as the parameter  $T$  increases. Here, the major drop occurs around  $T = 60min$ , which matches the day-ahead scheduling time step  $T_h$  described in Section IV-C. This is explained by the fact that the operating time scale of the load following reserve is  $T_h$  and

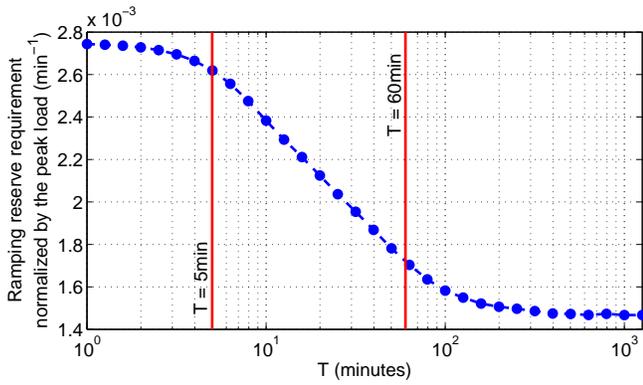


Fig. 5. The impact of geographical smoothing on ramping reserve requirement

eliminating the dynamics around that time scale also reduces the requirement for that type of reserve.

Finally, the impact of the geographical smoothing on the ramping reserve requirement is studied. The results are presented in Fig. 5. The results here also show that the ramping reserve requirement decreases as the parameter  $T$  increases. Eventually, the graph goes to saturation. However, while the ramping reserve operates at the day-ahead scheduling time scale  $T_h$ , the drop starts much earlier. This is explained by the fact that the ramping reserve requirement describes the ability of the system to quickly change its output and respond to high variability. Therefore, the calculations show that the variability appears as a multiplier of the ramping reserve requirement [11]. Thus, besides the smoothing around the characteristic time scale, the ramping reserve requirement is also directly affected by the overall drop of variability as the parameter  $T$  increases.

## VI. CONCLUSION AND FUTURE WORK

This paper studies the impact of the geographical distribution of wind turbines on variability and the requirements of three types of operating reserves, namely, load following, ramping and regulation. The results show that the best smoothing is achieved when all wind turbines have the same capacity and are uniformly distributed in the given area. The geographical smoothing reduces all three types of operating reserve requirements. The regulation reserve requirement starts to drop as the parameter  $T$  approaches the real-time balancing time step  $T_m$ , while reduction of the load following reserve requirement requires much wider distribution of the turbines when the parameter  $T$  approaches the day-ahead scheduling time step  $T_h$ . The ramping reserve requirement decreases for any drop of  $T$  parameter since it is directly related to the variability. While this study is focused on the wind generation, the proposed methodology can be extended to any type of renewable energy source. As a future work, the presented derivations are planned to extend to also include the forecast error.

## REFERENCES

- [1] E. A. DeMeo, W. Grant, M. R. Milligan, and M. J. Schuerger, "Wind plant integration: Costs, Status, and Issues," *Power and Energy Magazine, IEEE*, vol. 3, no. 6, pp. 38–46, 2005.
- [2] J. C. Smith, M. R. Milligan, E. A. DeMeo, and B. Parsons, "Utility Wind Integration and Operating Impact State of the Art," *Power Systems, IEEE Transactions on*, vol. 22, no. 3, pp. 900–908, 2007.
- [3] O. Edenhofer, L. Hirth, B. Knopf, M. Pahle, S. Schlömer, E. Schmid, and F. Ueckerdt, "On the economics of renewable energy sources," *Energy Economics*, vol. 40, Supple, no. 0, pp. S12–S23, Dec. 2013.
- [4] A. Muzhikyan, A. M. Farid, and K. Youcef-Toumi, "An Enterprise Control Assessment Method for Variable Energy Resource Induced Power System Imbalances. Part I: Methodology," *Industrial Electronics, IEEE Transactions on*, vol. 62, no. 4, pp. 2448–2458, 2015.
- [5] A. Muzhikyan, A. M. Farid, and K. Youcef-Toumi, "An Enterprise Control Assessment Method for Variable Energy Resource Induced Power System Imbalances. Part II: Parametric Sensitivity Analysis," *Industrial Electronics, IEEE Transactions on*, vol. 62, no. 4, pp. 2459–2467, 2015.
- [6] R. J. A. M. Stevens, D. F. Gayme, and C. Meneveau, "Effects of turbine spacing on the power output of extended wind-farms," *Wind Energy*, 2015.
- [7] I. Katic, J. Højstrup, and N. Jensen, "A Simple Model for Cluster Efficiency," in *EWEC'86. Proceedings. Vol. 1*, Rome, Italy, 1986, pp. 407–410.
- [8] N. Jensen, "A note on wind generator interaction," Risø National Laboratory, Tech. Rep., 1983.
- [9] N. G. Nygaard, "Wakes in very large wind farms and the effect of neighbouring wind farms," *Journal of Physics: Conference Series*, vol. 524, no. 012162, pp. 1–10, 2014.
- [10] A. Muzhikyan, A. M. Farid, and K. Youcef-Toumi, "An Enhanced Method for the Determination of Load Following Reserves," in *American Control Conference 2014 (ACC2014)*, Portland, OR, 2014, pp. 926–933.
- [11] A. Muzhikyan, A. M. Farid, and K. Youcef-Toumi, "An Enhanced Method for the Determination of the Ramping Reserves," in *American Control Conference 2015 (ACC2015)*, Chicago, Ill., 2015, pp. 994 – 1001.
- [12] A. Muzhikyan, A. M. Farid, and K. Youcef-Toumi, "An Enhanced Method for the Determination of the Regulation Reserves," in *American Control Conference 2015 (ACC2015)*, Chicago, Ill., 2015, pp. 1016 – 1022.
- [13] M. Milligan, K. Porter, E. DeMeo, P. Denholm, H. Holtinen, B. Kirby, N. Miller, A. Mills, M. O'Malley, M. Schuerger, and L. Soder, "Wind Power Myths Debunked," *Power and Energy Magazine, IEEE*, vol. 7, no. 6, pp. 89–99, 2009.
- [14] J. Apt, "The spectrum of power from wind turbines," *Journal of Power Sources*, vol. 169, no. 2, pp. 369–374, June 2007.
- [15] A. E. Curtright and J. Apt, "The character of power output from utility-scale photovoltaic systems," *Progress in Photovoltaics: Research and Applications*, vol. 16, no. 3, pp. 241–247, May 2008.
- [16] E. Ela, B. Kirby, E. Lannoye, M. Milligan, D. Flynn, B. Zavadil, and M. O'Malley, "Evolution of operating reserve determination in wind power integration studies," in *Power and Energy Society General Meeting, 2010 IEEE*, 2010, pp. 1–8.
- [17] H. Holtinen, M. Milligan, E. Ela, N. Menemenlis, J. Dobschinski, B. Rawn, R. J. Bessa, D. Flynn, E. Gomez-Lazaro, and N. K. Detlefsen, "Methodologies to Determine Operating Reserves Due to Increased Wind Power," *Sustainable Energy, IEEE Transactions on*, vol. 3, no. 4, pp. 713–723, 2012.
- [18] Bonneville Power Administration, "Wind Generation & Total Load in The BPA Balancing Authority." [Online]. Available: <http://transmission.bpa.gov/business/operations/wind/>