Static Resilience of Large Flexible Engineering Systems: Axiomatic Design Model & Measures

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***Abstract*—Our modern life has grown to depend on many and nearly ubiquitous large complex engineering systems. Many disciplines now seemingly ask the same question: "In the face of assumed disruption, to what degree will these systems continue to perform and when will they be able to bounce back to normal operation". This paper seeks to partially fulfill this need with *static* resilience measures for *large flexible engineering systems* based upon an axiomatic design model. This development is founded upon graph theory, and axiomatic design for large flexible engineering systems (LFESs). Central to the development is the concept of structural degrees of freedom as the available combinations of systems processes and resources which could be measured individually to describe system capabilities or measured sequentially to give a sense of the skeleton of a system's behavior. This approach facilitates the enumeration of service paths through a LFES along which valuable artifacts flow. Therefore, this work compares the value and quantity of service paths before and after a disruption as measures of static resilience – or survivability. To complete the contribution, a full illustrative example from the production system domain.**

***Index Terms*—resilience, reconfigurability, axiomatic design,large flexible engineering systems, graph theory, system architecture**

I. Introduction

Our modern life has grown to depend on many and nearly ubiquitous large complex engineering systems [1]. Transportation, water distribution, electric power, natural gas, healthcare, manufacturing and food supply are but a few. These systems are characterized by an intricate web of interactions within themselves [2] but also between each other [3]. Our heavy reliance on these systems coupled with a growing recognition that disruptions and failures; be they natural or man-made; unintentional or malicious; are inevitable. Therefore, in recent years, many disciplines have seemingly come to ask the same question: "How *resilient* are these systems?" Said differently, in the face of assumed disruption, to what degree will these systems continue to perform and when will they be able to bounce back to normal operation [4]. Furthermore, the major disruptions of 9/11, the 2003 Northeastern Blackout, and Hurricanes Katrina and Sandy has caused numerous agencies [5]–[7] to make resilient engineering systems a policy goal.

Naturally, a large body of academic literature has developed on the subject across multiple disciplines [4]. These include ecological [8], economic [9], organizational [10], network [11], socio-ecological [12] and psychological [13] resilience. Not surprisingly, a number of reviews [5], [13]–[15] on the topic have found that these contributions while complementary are not necessarily in agreement. The emerging field of resilience engineering, therefore, is still developing and requires formal quantitative definitions and frameworks [4], [5]. A key element to such rigorous approaches is the development of resilience measures which many, even recently, have identified as an area for concerted effort [4], [5], [13]–[19]. Such resilience measures would not only quantify resilience but could also inform designers and planners in advance how to best improve system resilience.

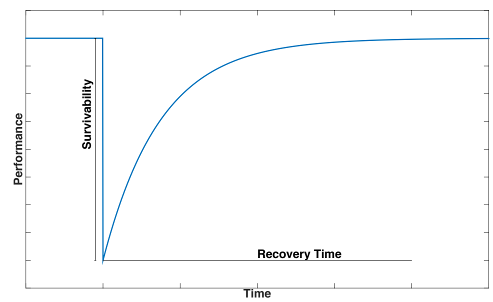

Fig. 1. Conceptual Representation of Resilient Performance

A. Contribution

This paper seeks to partially fulfill this need with *static* resilience measures for *large flexible engineering systems* based upon an axiomatic design model [20]. Thus, this work is in agreement with the developing consensus view which divides the life cycle property into two complementary aspects: a static "survival" property which measures the degree of performance after a disruption, and a dynamic "recovery" property which measures how quickly the performance returns to normal operation (Figure 1) [1], [4], [5], [13]–[19]. This paper also strikes a middle ground between two complementary approaches to resilience measurement. In the first, many resilience measures depend on traditional graph theoretic applications. The choice of axiomatic design over (traditional) graph theory allows the paper's scope to expand from *homo-functional* to *hetero-functional* systems. In the second, resilience measurement is conducted via complex simulation packages. While Axiomatic design allows for discrete-event system simulation, the model is at a higher level of abstraction to facilitate decision-making earlier in the system life-cycle. The paper's contribution generalizes previous work in which axiomatic design was also applied to production, transportation, water, and power systems [21]–[32]. One notable theme in the prior work was the enumeration of paths in these large flexible engineering systems which will be used here in the development of resilience measures. To demonstrate the broad applicability of the work, this paper inserts illustrative examples from transportation, power grid, water distribution, and production systems throughout.

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B. Scope & Terminology

This paper restricts its scope to the static resilience of large flexible engineering systems.

Definition 1. Large Flexible Engineering System (LFES) [20], [26], [27]: an engineering system with many functional requirements that not only evolve over time, but also can be fulfilled by one or more design parameters.

Here, a LFES is understood to constitute a physical system of engineered artifacts. The organizational system or network that designs, develops, and constructs, the LFES is explicitly excluded.

Although this paper relies on an Axiomatic Design model, it departs from its terminology without any change in interpretation. Instead, a set of definitions consistent with the INCOSE systems engineering handbook [33] are chosen. More specifically, the term "functional requirements" above is replaced from this point with "system processes" as therein defined [33]. These processes are physical and generic in nature, collectively constitute the system's function(s) and are consequently modeled in Section III with SysML activity diagrams. Given the system scope provided above, these processes should not be narrowly interpreted as technical, project, agreement, or organization processes as defined in the systems engineering handbook [33]. The term "design parameters" above is replaced from this point with "system resources" also as defined in the systems engineering handbook [33]. These system resources are utilized rather than consumed and constitute the system elements. This choice of terminology is entirely consistent with the prior works [21]–[31] upon which this paper is based.

C. Paper Outline

The measurement of resilience is naturally an indirect measurement process. Therefore, this paper paper follows a seven part discussion guided by Figure 2. Section II orients the discussion in terms of the two foundations of the work: graph theory and axiomatic design. Next, Section III introduces an Axiomatic Design model of "Structural Degree of Freedom" as a generalization of previous work [21]–[25] applied to production and transportation systems. Section IV then uses this model to develop static resilience measures for large flexible engineering systems. The developments are then demonstrated on an illustrative example in Section V. Section VI then provides a thorough discussion of the utility of the developments. Finally, Section VII concludes the work.

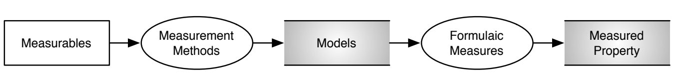


Fig. 2. A Generic Indirect Measurement Process [22], [34]

II. Background

This section summarizes the methodological foundations found in graph theory and axiomatic design in order to introduce the concept of "structural degrees of freedom" in the next section. Section II-A gives a brief introduction to graph theory while Section II-B introduces the application of axiomatic design to LFESs.

A. Graph Theory Introduction

As mentioned in the introduction, much of the existing resilience measurement literature has been based on graph theory [35], [36]. A number of basic definitions and theorems from this field are introduced for later in the development.

Definition 2. A graph [36]: $G = \{V, E\}$, consists of a collection of nodes V and a collection of edges E. Each edge $e \in E$ is said to join two nodes which are called its end points. If e joins $v_1, v_2 \in V$, we write $e = \langle v_1, v_2 \rangle$. Nodes v_1 and v_2, in this case, are said to be adjacent. Edge e is said to be incident with nodes v_1 and v_2 respectively.

Definition 3. A directed graph (digraph) [36]: G_D, consists of a collection of nodes V and a collection of arcs A, for which $G_D D = V, A$. Each arc $a = \langle v_1, v_2 \rangle$ is said to join node $v_1 \in V$ to another (not necessarily distinct) node v_2. Vertex v_1 is called the tail of a, whereas v_2 is its head.

Definition 4. Adjacency matrix [36]: A, is binary and of size $\sigma(V) \times \sigma(V)$ and its elements are given by

$$A(i,j) = \begin{cases} 1 & \text{if } \langle v_i, v_j \rangle \text{ exists} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where the operator $\sigma()$ gives the size of a set.

Theorem 1. *Number of Paths in a Graph [35]: The number of n-step paths between nodes i and j is given by* $A^N(i,j)$.

Theorem 2. *Number of Loops in a Graph [35]: The number of n-step loops from node i back to itself is given by* $A^N(i,i)$.

Definition 5. Diameter of a Graph [35]: The length of the longest geodesic (i.e shortest) path D between any nodes in a graph measured in number of steps.

While graph theory for decades has presented a useful abstraction across many applications, it has limitations from an engineering design and systems engineering perspective because it focuses primarily on an abstracted model of a system's form; neglecting an explicit description of system's function [37]. Consequently, it becomes challenging to link nodes and edges to physical variables as part of a detailed engineering design process. Also consider, Table I where Newman lists some common graph theory applications [35]. In all cases, the applications are large flexible *homo-functional* engineering systems where artifacts (of some kind) are transported between physical locations. While transportation processes that account for movement from one location to another are fundamentally different, ultimately they are of the same class or type. Thus, it is less than clear how graph theory may be applied to systems that are of a fundamentally transformative nature. As most engineering systems are *hetero-functional* (i.e. include functions of many types), graph theory may impede rigorous approaches where resilience can be engineered into the system.

TABLE I
Common Applications of Graph Theory [35]

Network	Node	Edge
Internet	Computer/Router	Wired/Wireless Data Connection
World Wide Web	Web Page	Hyperlink
Power Grid	Power or Substation	Transmission Line
Transportation	Intersections	Roads
Neural Network	Neuron	Synapse

B. Axiomatic Design for Large Flexible Engineering Systems

In contrast, axiomatic design for LFES (much like other engineering design methodologies [38]) overcomes many of these limitations because it specifically considers the allocation of system processes to system resources. This is achieved through the axiomatic design equation for LFESs [21]–[26]. Such processes and resources may be defined at any level of abstraction or decomposition at successive stages of engineering design.

$$P = J_S \odot R \tag{2}$$

where J_S is a binary matrix called a LFES "knowledge base", and $\odot$ is "matrix boolean multiplication" [21]–[26].

Definition 6. LFES Knowledge Base [21]–[26]: A binary matrix J_S of size $\sigma(P) \times \sigma(R)$ whose element $J_S(w, v) \in \{0, 1\}$ is equal to one when action e_{wv} exists as a system process $p_w \in P$ being executed by a resource r_v in R.

In other words, the system knowledge base itself forms a bipartite graph which maps the set of system processes to their resources.

III. Structural Degrees of Freedom in Axiomatic Design of Large Flexible Engineering Systems

In preparation for the development of static resilience measures in Section IV, this section now employs the axiomatic design knowledge base as a model for the development of structural degree of freedom measures in large flexible engineering systems. Consequently, the definitions found here are generalizations of those found in previous work where only production and transportation systems were considered [21]–[27]. Nevertheless, the interested reader is referred to these initial works for specific examples of each definition presented.

Following the indirect measurement process in Figure 2, the section proceeds in five parts: 1.) Description of Measurables 2.) Measurement Methods and three sets of related measures: 3.) Sequence-Independent structural degrees of freedom 4.) Sequence-dependent structural degrees of freedom 4.) Service-specific structural degrees of freedom.

A. Measurables

As mentioned in Section II-B, Axiomatic Design for Large Flexible Engineering Systems requires the definition of system resources and processes. These resources $R = M \cup B \cup H$ may be classified into transforming resources $M = \{m_1 \ldots m_{\sigma(M)}\}$, independent buffers $B = \{b_1 \ldots b_{\sigma(B)}\}$, and transporting resources $H = \{h_1 \ldots h_{\sigma(H)}\}$ [21]–[26]. The set of buffers $B_S = M \cup B$ is also introduced for later simplicity. Similarly, the high level system processes are formally classified into three varieties: transformation, transportation and holding processes.

Definition 7. Transformation Process [21]–[26]: A resource-independent, technology-independent process $p_{\mu j} \in P_\mu = \{p_{\mu 1} \ldots p_{\mu\sigma(P_\mu)}\}$ that transforms an artifact from one form into another.

Definition 8. Transportation Process [21]–[26]: A resource-independent process $p_{\eta u} \in P_\eta = \{p_{\eta 1} \ldots p_{\eta\sigma(P_\eta)}\}$ that transports artifacts from one buffer b_{sy_1} to b_{sy_2}. There are $\sigma^2(B_S)$ such processes of which $\sigma(B_S)$ are "null" processes where no motion occurs. Furthermore, the convention of indices $u = \sigma(B_S)(y_1 - 1) + y_2$ is adopted.

Definition 9. Holding Process [21]–[26]:A transportation independent process $p_{\varphi g} \in P_\varphi$ that holds artifacts during the transportation from one buffer to another.

These high level systems processes are defined to include both underlying physical function as well as the supporting enterprise control activities required for their operation [39].

Example 1. Table II [26] provides examples of transformation and transportation processes as well the three types of system resources. Holding processes are often introduced to differentiate between two transportation processes between an origin and a destination. In production systems, two material handler robots may have different end-effectors [21], [22], [24]. In water distribution systems, they can be introduced to differentiate between pipes that hold different types of water (e.g. potable and wastewater). In power grids, they can be used to differentiate transmission lines of different voltage level. In transportation systems, they have been used to differentiate between electrified and non-electrified roads [28].

TABLE II
SYSTEM PROCESSES & RESOURCES IN LFESS [26]

	P_μ	P_η	M	B	H
Production	Transformation	Transportation	Value-Adding Machines	Buffers	Material Handlers
Transportation	Entry/Exit	Transportation	Stations	Stations	Vehicles
Power Grids	Generation/ Consumption	Transmission	Generators/ Loads	Storage	Lines
Water	Extract/ Treat/ Pollute/ Dispose	Distribute	Treatment/ Demands	Storage	Lines

B. Measurement Methods

The measurables described in the previous subsection may be practically measured manually [40] or automatically [34] as has been done in a number of practical case studies [22], [40]. In the latter case, the LFES processes and resources are often found in IT-based management systems such as those found in the control centers of production shop floors, traffic operators, power grid operators, and water utilities. Figure 3, provides a SysML example of system processes, resources, and their allocation via swim lanes. Such an approach implicitly defines an ontological basis for the system's transformation and holding processes [22], [41].

C. Sequence-Independent Structural Degrees of Freedom

The concept of structural degrees of freedom was initially introduced on the basis of an analogy with mechanical degrees of freedom [21]–[26]. The heart of the concept rests in the realization that an allocated action $e_{wv} \in E$ (in the SysML activity diagram sense) [42] can be defined for each feasible combination of system process p_w and resource r_v. Naturally, the existence of e_{wv} in the LFES is represented by $J_S(w, v) = 1$. SysML classifies actions on the basis of whether they are *streaming* or *nonstreaming*. In the case of

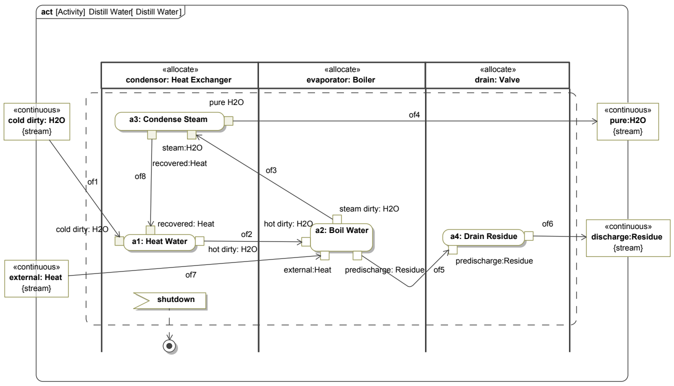


Fig. 3. Example: System Processes, Resources & Allocation [26]

the former, the allocated action would be described by a continuous time differential equation [43], [44]. In the case of the latter, the allocated action would be described as a discrete event [45]. With these notions in mind, reconfigurations and disruptions can add or remove these allocated actions or potentially reallocate a process to a resource.

The development of structural degrees of freedom continues with the introduction of a number constraints as is found in mechanical degrees of freedom. Here, the constraints are discrete and can apply in the operational time frame so as to eliminate actions from the action set. These constraints are said to be *scleronomic* as they are independent of action sequence. Such constraints can arise from any phenomenon that reduces the capabilities of a LFES e.g. resource breakdowns, inflexibly implemented processes and their control.

Definition 10. LFES Scleronomic Constraints Matrix [21]–[26]: A binary matrix K_S of size $\sigma(P) \times \sigma(R)$ whose element $K_S(w, v) \in \{0, 1\}$ is equal to one when a constraint eliminates event e_{wv} from the event set.

From these definitions of J_S and K_S, follows the definition of LFES sequence-independent structural degrees of freedom.

Definition 11. LFES Sequence-Independent Structural Degrees of Freedom [21]–[26]: The set of independent actions E_S that completely defines the available processes in a LFES. Their number is given by:

$$DOF_S = \sigma(E_S) = \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(R)} [J_S \ominus K_S](w, v) \quad (3)$$

$$= \sum_{w}^{\sigma(P)} \sum_{v}^{\sigma(R)} A_S(w, v) \quad (4)$$

In matrix form, Equation 3 can be rewritten in terms of the Frobenius inner product [46].

$$DOF_S = \langle J_S, \bar{K}_S \rangle_F = tr(J_S^T \bar{K}_S) \quad (5)$$

J_S and K_S can be reconstructed straightforwardly from smaller knowledge bases that individually address transformation, transportation and holding processes. $P_\mu = J_M \odot M$, $P_\eta = J_H \odot R$, $P_\gamma = J_\gamma \odot R$. In this case, it follows that [21]–[26]:

$$J_S = \left[\begin{array}{c|c} J_M & \mathbf{0} \\ \hline \multicolumn{2}{c}{J_{\bar{H}}} \end{array} \right] \quad (6)$$

$$K_S = \left[\begin{array}{c|c} K_M & \mathbf{0} \\ \hline \multicolumn{2}{c}{K_{\bar{H}}} \end{array} \right] \quad (7)$$

where [21]–[26]

$$\begin{aligned} J_{\bar{H}} &= \left[J_\varphi \otimes \mathbf{1}^{\sigma(P_\eta)} \right] \cdot \left[\mathbf{1}^{\sigma(P_\varphi)} \otimes J_{\bar{H}} \right] \\ K_{\bar{H}} &= \left[K_\varphi \otimes \mathbf{1}^{\sigma(P_\eta)} \right] \cdot \left[\mathbf{1}^{\sigma(P_\varphi)} \otimes K_{\bar{H}} \right] \end{aligned} \quad (8)$$

and $\mathbf{1}^n$ is a ones vector of length n. Consequently, transformational (DOF_M) and transportational (DOF_H) degree of freedom measures may be calculated as shown in Table III [21]–[26].

TABLE III
TYPES OF LFES SEQUENCE-INDEPENDENT DEGREE OF FREEDOM MEASURES [21]–[26]

Measure	Process Element	Resource Element	Knowledge Base	Constraint Matrix	Measure Function
DOF_M	$p_{\mu j}$	m_k	J_M	K_M	$\langle J_M, \bar{K}_M \rangle_F$
DOF_H	$p_{\eta u}$	r_v	J_H	K_H	$\langle J_H, \bar{K}_H \rangle_F$
$DOF_{\bar{H}}$	$P_{\gamma\varrho}$	r_v	$J_{\bar{H}}$	$K_{\bar{H}}$	$\langle J_{H\gamma}, \bar{K}_{H\gamma} \rangle_F$
DOF_S	p_w	r_v	J_S	K_S	$\langle J_{S\gamma}, \bar{K}_{S\gamma} \rangle_F$

Intuitively, the sequence-independent structural degrees of freedom measure the number of ways that all of the system processes may be executed. They provide a flexible expression of LFES capabilities in the design and operational phases. From an axiomatic design perspective, the usage of knowledge bases facilitates further detailed engineering design [20]. The constraints matrix captures the potential for resource breakdowns and inflexibly implemented processes either physically or informatically in associated control and management structures. Additionally, from a graph theory perspective, the knowledge base forms a bipartite graph which may experience node or edge addition or elimination during operation. In the next section, a graph will be formed with structural degrees of freedom representing the nodes.

D. Sequence-Dependent Structural Degrees of Freedom

The previous subsection recalled the development of sequence-independent structural degrees of freedom. A LFES, however, has constraints that introduce dependencies in the sequence of actions. A new measure is required for the sequence-dependent capabilities of the LFES [21]–[26].

Example 2. Reconsider Figure 3 [26]. Each solid line between activities represents a feasible sequence or pair of activities. Unconnected activities effectively have a design constraint that prohibits their sequential operation.

Definition 12. LFES Sequence-Dependent Structural Degrees of Freedom [21]–[26]: The set of independent pairs of actions $z_{\psi_1\psi_2} = e_{w_1v_1}e_{w_2v_2} \in Z$ of length 2 that completely describe the system language. The number is given by:

$$DOF_\rho = \sigma(\mathcal{Z}) = \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} [J_\rho \ominus K_\rho](\psi_1, \psi_2) \quad (9)$$

$$= \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} [A_\rho](\psi_1, \psi_2) \quad (10)$$

where J_ρ and K_ρ are defined below.

TABLE IV
TYPES OF SEQUENCE-DEPENDENT PRODUCTION DEGREE OF FREEDOM MEASURES [24]–[26]

Type	Measures	Processes	Resources	Knowledge Base	Constraint Matrix	Perpetual Constraint	Measure Function
I	$DOF_{MM\rho}$	$P_\mu P_\mu$	M, M	$J_{MM\rho} = \left[J_M \cdot \bar{K}_M\right]^V \left[J_M \cdot \bar{K}_M\right]^{VT}$	$K_{MM\rho}$	$K_1 = K_2$	$\langle J_{MM\rho}, \bar{K}_{MM\rho}\rangle_F$
II	$DOF_{MH\rho}$	$P_\mu P_\eta$	M, R	$J_{MH\rho} = \left[J_M \cdot \bar{K}_M\right]^V \left[J_H \cdot \bar{K}_H\right]^{VT}$	$K_{MH\rho}$	$k_1 - 1 = (u_1 - 1)/\sigma(B_S)$	$\langle J_{MH\rho}, \bar{K}_{MH\rho}\rangle_F$
III	$DOF_{HM\rho}$	$P_\eta P_\mu$	R, M	$J_{HM\rho} = \left[J_H \cdot \bar{K}_H\right]^V \left[J_M \cdot \bar{K}_M\right]^{VT}$	$K_{HM\rho}$	$k_1 - 1 = (u_1 - 1)\&\sigma(B_S)$	$\langle J_{HM\rho}, \bar{K}_{HM\rho}\rangle_F$
IV	$DOF_{HH\rho}$	$P_\eta P_\eta$	R, R	$J_{HH\rho} = \left[J_H \cdot \bar{K}_H\right]^V \left[J_H \cdot \bar{K}_H\right]^{VT}$	$K_{HH\rho}$	$(u_1-1)\%\sigma(B_S) = (u_2 - 1)/\sigma(B_S)$	$\langle J_{HM\rho}, \bar{K}_{HM\rho}\rangle_F$
ALL	DOF_ρ	PP	R, R	$J_\rho = \left[J_S \cdot \bar{K}_S\right]^V \left[J_S \cdot \bar{K}_S\right]^{VT}$	K_ρ	All of the Above	$\langle J_{HM\rho}, \bar{K}_{HM\rho}\rangle_F$

Definition 13. LFES Rheonomic knowledge base [24]–[26]: A square binary matrix J_ρ of size $\sigma(P)\sigma(R) \times \sigma(P)\sigma(R)$ whose element $J_\rho(\psi_1, \psi_2) \in \{0, 1\}$ is equal to one when string z_{ψ_1,ψ_2} exists. It may be calculated directly as

$$J_\rho = \left[J_S \cdot \bar{K}_S\right]^V \left[J_S \cdot \bar{K}_S\right]^{VT} \tag{11}$$

where $()^V$ is shorthand for vectorization (i.e. vec()).

Definition 14. LFES Rheonomic Constraints Matrix K_ρ [24]–[26]: a square binary constraints matrix of size $\sigma(P)\sigma(R) \times \sigma(P)\sigma(R)$ whose elements $K(\psi_1, \psi_2) \in \{0, 1\}$ are equal to one when string $z_{\psi 1 \psi 2} = e_{w_1 v_1} e_{w_2 v_2} \in Z$. is eliminated and where $\psi = \sigma(P)(v - 1) + w$.

Unlike its scleronomic counterpart where a zero matrix is possible, the rheonomic production constraints matrix has the perpetually binding constraints described in Table IV. These ensure that the origin and destination of consecutive events match. Accurately keeping track of these constraints simultaneously is challenging. The final calculation of these minimal constraints is most easily implemented in a scalar fashion using FOR loops while adhering to the following relationships of indices. $\psi = \sigma(P)(v - 1) + w$. $v = k$ $\forall k = [1 \ldots \sigma(M)]$. $w = [\sigma(P_\eta)(g - 1) + u] + j$ [24], [26].

As with sequence-independent structural degrees of freedom, sequence-dependent structural degrees of freedom may be classified in terms of their transformational and transportational variants. The calculation of the four types of measures is summarized in Table IV and maintains an intuitive symmetry [24]. In practice, the formation of the associated constraints matrices $K_{MM\rho}$, $K_{MH\rho}$, $K_{HM\rho}$,$K_{HH\rho}$ is an extra computational expense if K_ρ has already been formed. Instead, the associated rheonomic production degree of freedom measures can be calculated by the appropriate replacement of J_M or $J_{\bar{H}}$ with a zero matrix in Equation 6 [24].

Intuitively, the sequence-dependent structural degrees of freedom measure the number of ways that pairs of system processes may be executed. In other words, the system language $\mathbb{L}$ can be described equally well in terms of the Kleene closure [45] of the sequence-independent and sequence-dependent structural degrees of freedom.

$$\mathbb{L} = E_S^* = Z^* \tag{12}$$

From a graph theory perspective, Equation 11 shows that the sequence independent structural degrees of freedom are explicitly vectorized (as is commonly done with mechanical degrees of freedom). Furthermore, boolean difference of the rheonomic knowledge base and constraints matrix forms an adjacency matrix A_ρ between nodes defined as structural degrees of freedom.

E. Service-Specific Structural Degrees of Freedom

While it is important to quantify the capabilities of a LFES, it is even more important to assess how well these capabilities are matched to the services that it intends to offer. This is especially important in the context of resilience measurement where either the required services or the existing capabilities may be changed. Furthermore, resilience is often measured with respect to a certain performance property [47]; thus implicitly defining one or more services of interest. This subsection builds upon the efforts of the previous sections to develop service-specific structural degrees of freedom. Intuitively speaking, the required services select out the sequence-independent structural degrees of freedom and the mathematical form of the associated measure is developed on that basis. To begin, the set of services is simplistically modeled so that the service-specific structural degree of freedom measures may be developed later.

1) Service Modeling: Before a treatment of service-specific structural degrees of freedom can be initiated, a systematic approach to describing services is required. For the sake of simplicity, the scope of this work restricts itself to a class of services that do not require the mixing/assembly of heterogeneous artifacts, or the disjoining/separation of the same. The interested reader is referred to [22] for work that address such types of services.

A LFES may provide a set of services $L = \{l_1, \ldots, l_{\sigma(L)}\}$ where each service l_i has its associated set of service activities $e_{xl_i} \in E_{l_i}$ which when all are completed result in the delivery of the service.

Definition 15. Service Activity [22], [24], [26]: A specific transformation process that may be applied as a part of larger service.

Consequently, the delivery of the service is described as a sequence of service activities [22], [24], [26]:

$$z_{l_i} = e_{x_1 l_i} e_{x_2 l_i} \ldots e_{x_{\sigma(E_{l_i})} l_i} \tag{13}$$

Example 3. Table V provides concrete examples of services for production, transportation, power grid and water distribution systems.

These examples offer a number of fundamental insights. First, it is important to note that this work implicitly assumes

TABLE V
EXAMPLE SYSTEM SERVICES IN LFESS [26]

Transportation:	{Enter passenger at the origin station, Exit the passenger at the destination}
Power Grid:	{Generate electricity at the origin, Consume the electricity at the destination}
Water Distribution:	{Extract the water at the origin, Treat the water, Degrade the water through use, Dispose of the water at the destination}
Production:	{Enter the part to an input buffer, Mill the part, Drill a hole in the part, Polish the part, Exit the part from an output buffer}

that the LFES is an *open* system. As such all of the services described above begin with an *entrance* of some artifact and end with its *exit*. Furthermore, water distribution and production systems require more careful thought given the potential heterogeneity of transformation processes and their artifacts. Finally, all of the systems do not fundamentally require transportation processes because all transformation processes required by the service could conceivably be realized in one location. This generalization is true even for transportation systems for the trivial case that the passenger's desired origin and destination are the same.

2) Service Activity Feasibility: The feasibility of a given service on an activity-by-activity basis follows straightforwardly from the introduction of two feasibility matrices.

Definition 16. Service Transformation Feasibility Matrix $\Lambda_{\mu i}$ [22], [24], [26]: For a given service l_i, a binary matrix of size $\sigma(E_{l_i}) \times \sigma(P_\mu)$ whose value $\Lambda_{\mu i}(x,j) = 1$ if e_{xl_i} realizes transformation process $p_{\mu j}$.

Definition 17. Service Transportation Feasibility Matrix $\Lambda_{\gamma i}$ [22], [24], [26]: A binary row vector of size $1 \times \sigma(P_\gamma)$ whose value $\Lambda_{\gamma i}(g) = 1$ if product l_i can be held by holding process $p_{\gamma g}$.

Note that because the service model only includes activities of a transformative nature, $\Lambda_{\gamma i}$ only marks feasibility between the service as a whole and the set of holding processes [22], [24].

3) Calculation of Service-Specific Structural Degrees of Freedom: From these definitions, it is straightforward to measure the number of LFES service-specific structural degrees of freedom. $\Lambda_{\mu i}$ and $\Lambda_{\gamma i}$ must first be used to produce a number of "selector" matrices that are equal in size to their corresponding LFES (scleronomic) knowledge base. Which one is used depends on the scope of interest; be it the type of process (e.g. transformation or transportation) or the desired scale (i.e. a single service activity, a whole service, or the whole set of services). Table VI summarizes the definition and formulation of the multiple types of service selector matrices [24], [26].

From these definitions, it is straightforward to measure the number of LFES service-specific structural degrees of freedom [24], [26].

$$DOF_{LS} = \langle \Lambda_{SL} \cdot J_S, \bar{K}_S \rangle_F \tag{14}$$

As mentioned at the beginning of the section, this intuitive form of service-specific degrees of freedom shows that the services effectively select out the structural degrees of freedom provided by the LFES [24], [26].

TABLE VI
TYPES OF SERVICE SELECTOR MATRICES [22], [24], [26]

Symbol	Formula	Scope
Λ_{Mxi}	$\left[e_x^T \Lambda_{\mu i}\right]^T \mathbf{1}^{\sigma(M)T}$	Service Activity – Transformation
Λ_{Mi}	$\left[\bigvee_x^{\sigma(E_L)} \Lambda_{\mu i}\right]^T \mathbf{1}^{\sigma(M)T}$	Service – Transformation
Λ_{ML}	$\left[\bigvee_i^{\sigma(L)} \bigvee_x^{\sigma(E_L)} \Lambda_{\mu i}\right]^T \mathbf{1}^{\sigma(M)T}$	Service Line – Transformation
Λ_{Hi}	$\left[\Lambda_{\gamma i} \otimes \mathbf{1}^{\sigma(P_\eta)T}\right]^T \mathbf{1}^{\sigma(R)T}$	Service – Transportation
Λ_{Hi}	$\left[\left[\bigvee_i^{\sigma(L)} \Lambda_{\gamma i}\right] \otimes \mathbf{1}^{\sigma(P_\eta)T}\right]^T \mathbf{1}^{\sigma(R)T}$	Service – Transportation
Λ_{Sxi}	$\left[\begin{array}{c\|c} \Lambda_{Mxi} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{Hi}} \end{array}\right]$	Service Activity – Transformation & Transportation
Λ_{SMxi}	$\left[\begin{array}{c\|c} \Lambda_{Mxi} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\mathbf{0}} \end{array}\right]$	Service Activity – Transformation
Λ_{Si}	$\left[\begin{array}{c\|c} \Lambda_{Mi} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{Hi}} \end{array}\right]$	Service – Transformation & Transportation
Λ_{SHi}	$\left[\begin{array}{c\|c} \mathbf{0} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{Hi}} \end{array}\right]$	Service – Transportation
Λ_{SL}	$\left[\begin{array}{c\|c} \Lambda_{ML} & \mathbf{0} \\ \hline \multicolumn{2}{c}{\Lambda_{HL}} \end{array}\right]$	Service Line – Transformation & Transportation

F. Relevance of Structural Degrees of Freedom to Resilience Measurement

The structural degree of freedom measures have a direct relevance to resilience measurement. This is because they provide a quantitative description of the system "reconfiguration" potential [21], [34], [34]; be it an arbitrary disruption or restoration of the system's capabilities. Such a reconfiguration processes would be mathematically described by [21], [24], [26], [48]:

$$(J_{S_o}, K_{S_o}, K_{\rho_o}) \rightarrow (J_S, K_S, K_\rho) \tag{15}$$

The static resilience or survivability measures developed in the next section depend on the performance degradation arising from the reconfiguration process described by Equation 15 and implicitly assume that $DOF_{\rho_o}(J_{S_o}, K_{S_o}, K_{\rho_o}) \geq DOF_\rho(J_S, K_S, K_\rho)$. In that sense, it assumes that (J_S, K_S, K_ρ) are associated with closed sets of processes and resources. That said, future work on dynamic resilience or "recovery" can loosen this assumption and allow for an expansion in these sets to achieve greater resilience.

IV. DEVELOPMENT OF STATIC RESILIENCE MEASURES

The background provided by the previous two sections allow for the development of static resilience measures for large flexible engineering systems. Many resilience measures in the existing literature are based upon some form of calculation of the shortest path length through a traditional graph [49]–[52]. Similarly, the static resilience measures proposed in this work depend on paths through a graph as well. In contrast, however, the measures derive from the *number* rather than the *length* of those paths. Furthermore, the paths are through a graph based upon nodes defined as structural degrees of freedom rather than simply locations as is often done in traditional graph theory. This section proceeds in two parts. Section IV-A enumerates the number of paths for a service offered by an LFES. Section IV-B then proposes the static resilience measures.

A. Path Enumeration in Large Flexible Engineering Systems

The development of static resilience measures depends on the enumeration of the number of feasible paths for each of the services provided by the LFES. The first step in this calculation is the translation of the service string in Equation 13 to an equivalent string composed of structural degrees of freedom. Because the service feasibility of transformation and transportation processes is fundamentally different, the new equivalent string must also alternate between the two types of degrees of freedom [22], [24], [27].

$$z = e_{\mu_{j_1} m_{k_1}} [e_{\gamma_{\varphi_1} r_{v_1}}]^{*D} e_{\mu_{j_2} m_{k_2}} [e_{\gamma_{\varphi_2} r_{v_2}}]^{*D} \ldots e_{\mu_{j_{\sigma(E_{l_i})}} m_{k_{\sigma(E_{l_i})}}} \quad (16)$$

where the exponent of an activity is used to denote the number of times it is repeated, and $*D$ denotes the repetition between zero and D times where D is the diameter of the transportation network of the LFES as defined in Definition 5. For simplicity, Equation 16 can be rewritten explicitly in terms of its repeating pattern:

$$\left[e_{\mu_{j_x} m_{k_x}} [e_{\gamma_{\varphi_1} r_{v_1}}]^{*D} e_{\mu_{j_2} m_{k_2}} \right]^{\sigma(E_{l_i})} \quad (17)$$

Example 4. The string in Equation 17 is overly general for most LFESs [27]. It is useful to specialize it for each of the four example LFESs discussed in this work:

Transportation [27]: The passenger must enter at the origin station, take a number of transportation degrees of freedom and then leave at the destination. Therefore, Equation 16 can collapse to:

$$e_{\mu_{j_1} m_{k_1}} [e_{\gamma_{\varphi_1} r_{v_1}}]^{*D} e_{\mu_{j_2} m_{k_2}} \quad (18)$$

Power Grid [27]: The electricity must be generated at a power plant, pass through a number of power lines and then be consumed at a destination. Therefore, power grids also follow the string in Equation 18.

Production [27]: Production system's naturally minimize transportation degrees of freedom because they are viewed as non-value adding. Good production system design practice allows at most one transportation process between any two transformation degrees of freedom. Therefore, Equation 17 can collapse to [27]:

$$\left[e_{\mu_{j_x} m_{k_x}} [e_{\gamma_{\varphi_1} r_{v_1}}] e_{\mu_{j_2} m_{k_2}} \right]^{\sigma(E_{l_i})} \quad (19)$$

Note that this string still encompasses the potential for successive transformation events because the transportation events include "null processes" where no motion occurs at a system resource.

Water Distribution [27]: Taken alone, the potable water distribution network resembles the transportation and power grid systems as in Equation 18. However, when the water distribution system is taken to include storm drains, wastewater, and recycled water, then the full string in Equation 17 is required.

The next step in path enumeration is to rewrite Equation 17 in terms of sequence-dependent structural degrees of freedom using Equation 12 [27].

$$\left[z_{\psi_{M_x} \psi_H} [z_{HH}]^{*D-1} z_{\psi_{M_{x+1}} \psi_H} \right]^{\sigma(E_{l_i})-1} \quad (20)$$

where $z_{\psi_{M_x} \psi_H}$, z_{HH}, and $z_{\psi_{M_{x+1}} \psi_H}$ are Type II, III and IV sequence-dependent structural degrees of freedom respectively (See Table IV). This relatively compact form explicitly states in terms of the sequence-dependent structural degrees of freedom the paths through a LFES that would make a given service feasible.

The enumeration of these paths is straightforward using Theorem 1. To that effect, an adjacency matrix is written for each of the three types of sequence dependent structural degrees of freedom in Equation 20. This requires the application of Equation 9 as specialized by Tables III, IV, and VI. For any given service i [27],

$$A_{MHx_1\rho} = \left[\Lambda_{SMx_1} \cdot J_{SM} \cdot \bar{K}_{SM} \right]^V \left[\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_{SH} \right]^{VT} \ominus K_\rho \quad (21)$$

$$A_{HH\rho} = \left[\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_{SH} \right]^V \left[\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_{SH} \right]^{VT} \ominus K_\rho \quad (22)$$

$$A_{HMx_2\rho} = \left[\Lambda_{SH} \cdot J_{SH} \cdot \bar{K}_{SH} \right]^V \left[\Lambda_{SMx_2} \cdot J_{SM} \cdot \bar{K}_{SM} \right]^{VT} \ominus K_\rho \quad (23)$$

where x_1 and x_2 are the indices of sequential service activities. $x_2 = x_1 + 1$. These adjacency matrices are then multiplied together following Equation 20 to give the number of paths through a LFES for a given service i

$$DOF_{P_i} = \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} \left[A_{P_i} \right](\psi_1, \psi_2) \quad (24)$$

where

$$A_{P_i} = \sum_{d=1}^{\mathcal{D}} \left[(A_{MHx\rho}) \left(A_{HH\rho}^{d-1} \right) (A_{HMx+1\rho}) \right]^{\sigma(E_{l_i})-1} \quad (25)$$

Note that the summation over d is required here because their may exist transportation paths of length one up to $\mathcal{D} - 1$ between successive transformations [25].

Example 5. As expected, this formula is overly general for most LFESs [27]. It is useful to specialize it for each of the four example LFESs discussed in this work:

Transportation/Power/Potable Water Distribution [27]:

$$A_{P_i} = \sum_{d=1}^{\mathcal{D}} \left[(A_{MHx\rho}) \left(A_{HH\rho}^{d-1} \right) (A_{HMx+1\rho}) \right] \quad (26)$$

which confirms the result in [25].

Production [27]:

$$A_{P_i} = \left[(A_{MHx\rho}) (A_{HMx+1\rho}) \right]^{\sigma(E_{l_i})-1} \quad (27)$$

which confirms the result in [24].

Remark. For the application of resilience measurement, it is often important to distinguish between the number of transportation paths and the number of *simple* transportation paths (i.e. without loops). Intuitively speaking, transportation loops represent a type of unuseful redundancy. For this purpose, it may be necessary to eliminate the number of loops from the calculation of Equation 25. This can be done straightforwardly by eliminating the diagonal at each power of $A_{HH\rho}$. Instead $A_{HH\rho}^{D-1}$ can be replaced with $A_{HHS\rho}^{D-1}$ which is defined recursively [27]:

$$A^n_{HHS\rho} = \left[A^{n-1}_{HHS\rho} - diag(A^{n-1}_{HHS\rho})\right][A_{HHS\rho}] \quad (28)$$
$$A_{HHS\rho} = A_{HH\rho} - diag(A_{HH\rho})$$

B. Static Resilience Measures

Returning to Figure 1, this work is concerned with *static* resilience measures which effectively quantifies *survivability* or the level of immediate degradation caused by a disruption represented as a change in (J_S, K_S,K_ρ). Furthermore, the performance of the service depends on the existence of a *complete* path for its realization. Therefore, a static resilience measure can be defined as a function of the existence or number of paths that realize a service which are in turn a function of the structural degrees of freedom. The section proceeds by defining the concept of performance then proposing two resilience measures.

1) Definition of Performance: In the context of this work, the engineering performance of an LFES depends purely on its static structure. Consider a service i, that begins with DOF e_{ψ_1} and ends with e_{ψ_2}. It delivers a quantity $Q_i(\psi_1, \psi_2)$ worth of a valuable artifact. The performance of that service is given by [27]:

$$EP_i = \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} Q_i(\psi_1, \psi_2) \cdot bi\left[A_{Pi}(\psi_1, \psi_2)\right] \quad (29)$$

where $bi()$ is the binary function that returns 1 for all positive quantities and zero otherwise. Here, it is assumed that the number of paths for the service is not as important as the simple existence of such a path. As such, it assumes that each path is not capacity limited in Q. For the full engineering performance of the LFES over all services, it is necessary to linearly combine this measure with those of the other services [27].

$$EP = \sum_{i}^{\sigma(L)} \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} c_i Q_i(\psi_1, \psi_2) \cdot bi\left[A_{Pi}(\psi_1, \psi_2)\right] \quad (30)$$

where c_i is a measure of value of the i^{th} service such as utility, cost or profit that harmonizes the units of all Q_i.

In the context of this work, it is also useful to write the engineering performance measure explicitly in terms of the knowledge base and constraint matrices (J_S, K_S, K_ρ). With the change of notation $A_{Pi}(\psi_1, \psi_2) = A_{P_i\psi_1\psi_2}$ [27],

$$EP = \sum_{i}^{\sigma(L)} \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} c_i Q_i(\psi_1, \psi_2) \cdot bi\left[A_{P_i\psi 1\psi 2}(J_S, K_S, K_\rho)\right] \quad (31)$$

Because the LFES's dynamics in terms of constitutive, continuity and compliance relations have not been modeled, Q_i is modeled as a constant rather than as a function of the structural degrees of freedom. Nevertheless, the ability to provide Q_i does require the existence of at least one service path and so studying the presence of such a path is a logical first step.

2) Actual Resilience: The actual engineering resilience (AER) with respect to a particular disruption that takes the system through the transformation: $(J_{S_o}, K_{S_o}, K_{\rho_o}) \rightarrow (J_S, K_S, K_\rho)$ can now be defined straightforwardly [27].

$$AER = \frac{EP\left(J_S, K_S, K_\rho\right)}{EP\left(J_{S_o}, K_{S_o}, K_{\rho_o}\right)} \quad (32)$$

This actual resilience measure benefits from the binary function $bi()$. As expected, LFESs that exhibit some path redundancy for their services will not suffer from performance degradation. That said, the $bi()$ also hides the effect of redundancy elimination caused by successive disruptions and so is not the most accurate predictor of the LFES' "health" towards future disruptions.

3) Latent Engineering Resilience: To address the limitations of the actual resilience measure, a latent engineering resilience measure is proposed [27]. LER=

$$\frac{\displaystyle\sum_{i}^{\sigma(L)} \sum_{\psi_1}^{\sigma(E_S)} \sum_{\psi_2}^{\sigma(E_S)} c_i Q_i(\psi_1, \psi_2) \cdot \frac{[A_{P_i\psi 1\psi 2}(J_S, K_S, K_\rho)]}{[A_{P_i\psi 1\psi 2}(J_{S_o}, K_{S_o}, K_{\rho_o})]}}{EP\left(J_{S_o}, K_{S_o}, K_{\rho_o}\right)} \quad (33)$$

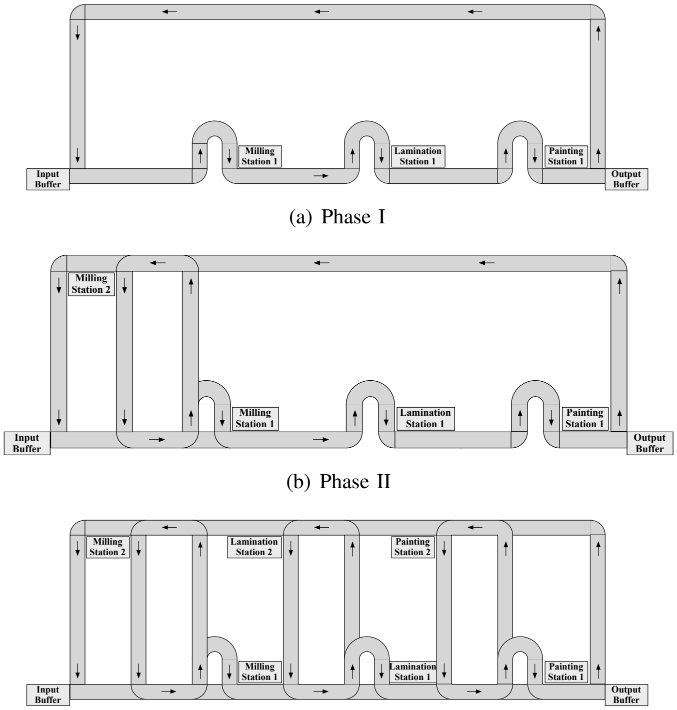
Here, the LER measure degrades gracefully with the transformation $(J_{S_o}, K_{S_o}, K_{\rho_o}) \rightarrow (J_S, K_S, K_\rho)$ because of the ratio of actual enumerated paths to the prior enumerated paths. It may be used to show how a given disruption eliminates some but not all of the paths of a given service.

V. Illustrative Example

Based on the discussion of the previous section, a large flexible engineering system with several types of functions is chosen for an illustrative example. This choice demonstrates the analytical capability of the proposed measures relative to those applicable to homo-functional systems. As mentioned in Examples 3 and 4, production systems provide an extra level of functional heterogeneity that is less apparent in transportation, electric power and potable water distribution. To this end, the "Starling Manufacturing System" is taken as the test case for its functional heterogeneity and redundancy and its resource flexibility while maintaining a moderate size [21], [22], [24], [26], [27].

The system produces customized bird feeders from cylindrical wooden components. In this example, customers can choose between bird feeders of red and yellow color, and small and large radii. The wooden cylinders are turned for slots and tabs, milled, laminated, and painted. All product variants have an injection molded dome roof and a base which doubles as a bird perch. These components are manually snapped onto the cylindrical bird feeders after production and are not further discussed in this example.

The production system itself is considered in three configurations that includes machining, lamination, and painting machines. Figure 4(a) shows the initial configuration, Figure 4(b) adds a second machining station, and Figure 4(c) makes all three value-adding resources redundant. Two shuttles transport the cylinders between the machines and buffer. The first has a flexible fixture which accommodates both radius sizes while the second can only carry cylinders of small radius.



(a) Phase I

(b) Phase II

(c) Phase III

Fig. 4. Starling manufacturing system in three configurations

Next, the following sets of processes and resources are identified. In Phase I, M={Machining Station 1, Lamination Station 1, Painting Station 1}. B={Input Buffer, Output Buffer}. H={Shuttle A, Shuttle B}. P_M={Lathe Tab, Lathe Slot, Mill Hole, Laminate, Paint Red, Paint Yellow}. $P_\eta = \{m_i m_j, m_i b_k, b_k m_i, b_k b_l\} \forall i, j = 1, 2, 3, k, l = 1, 2$. P_γ={Small Radial, Big Radial, Axial}. The production processes and resources for the other system configurations may be determined analogously. Figure 5 presents the transformation, transportation, and holding knowledge bases for the Starling Manufacturing System in Phase I as monochrome images. It also shows the service transformation and transportation feasibility matrices for the small red bird feeder. For the sake of simplicity in the discussion, all four types of bird feeders are taken to sell the same quantities at the same price. The scleronomic constraints matrices are initially set to zero. The rheonomic constraints matrix has the minimal constraints identified in Section III-D. The knowledge bases, constraints matrices, and service feasibility matrices for the other production system configurations and product variants can be readily formed by analogy. In order to measure the resilience of the production system, three types of disruptions are considered: a broken tab-lathing tool in Machining Station 1, a fault in Lamination Station 1, and a fault in Shuttle A which is capable of holding both small and large bird feeders.

The production paths for the three production system configurations under the various disruptions are shown in Table VII. The corresponding values of actual and latent resilience are shown in Table VIII. Under normal operation, the number of production paths greatly expands as more redundancy is introduced into the system between Phases I and III. However, the utility of this redundancy is entirely different under

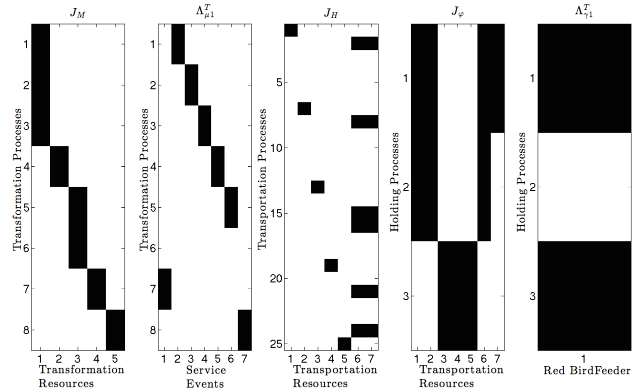


Fig. 5. Inputs for Phase I Starling Manufacturing System with a Small Red Bird Fedder: a.)Transformation Knolwedge Base b.) Service Transformation Feasibility Matrix c.)Transportation Knowledge base d.)Holding Knowledge Base e.) Service Transportation Knowledge Base

different types of disruptions.

TABLE VII
PRODUCTION PATHS OF PRODUCT LINE FOR VARIOUS FAULTS

	Normal Stage 1	Broken Tool	Faulted Lamination	Faulted Shuttle A
Small Red	16	0	0	1
Small Yellow	16	0	0	1
Large Red	1	0	0	0
Large Yellow	1	0	0	0
	Normal Stage 2	Broken Tool	Faulted Lamination	Faulted Shuttle A
Small Red	96	80	0	4
Small Yellow	96	80	0	4
Large Red	4	3	0	0
Large Yellow	4	3	0	0
	Normal Stage 3	Broken Tool	Faulted Lamination	Faulted Shuttle A
Small Red	1152	576	576	32
Small Yellow	1152	576	576	32
Large Red	32	16	16	0
Large Yellow	32	16	16	0

TABLE VIII
ACTUAL & LATENT RESILIENCE VALUES FOR VARIOUS FAULTS

	Normal Stage 1	Broken Tool	Faulted Lamination	Faulted Shuttle A
AER	1	0	0	1/2
LER	1	0	0	1/32
	Normal Stage 2	Broken Tool	Faulted Lamination	Faulted Shuttle A
AER	1	1	0	1/2
LER	1	19/24	0	1/48
	Normal Stage 3	Broken Tool	Faulted Lamination	Faulted Shuttle A
AER	1	1	1	1/2
LER	1	1/2	1/2	2/189

In Phase I, under normal operation, the small and large bird feeders exhibit a difference in the number of production paths because the latter has two available shuttles between transformation processes while the latter only has one. Thus, when Shuttle A fails, all of the large bird feeders can not be produced while the small bird feeders continue on Shuttle B. As expected, the 50% drop in AER reflects that 50% of the birdfeeder types can no longer be produced. In contrast, the corresponding LER measure has fallen to 0.03125 (i.e. 1/32). This result shows, that in reality, from a "system health" perspective, the disruption of Shuttle A eliminated production paths for all product variants; leaving only one for each type of bird feeder. Such a dedicated manufacturing line is highly brittle; and this is further shown by complete elimination of all production paths for the two other types of disruptions.

In Phase II, an additional milling station provides redundancy for three transformation processes: lathe tab, lathe slot, and mill hole. Setting aside the transportation system redundancy, the number of paths in normal operation should grow by $2^3 = 8$. However, it only grows by a factor of 4 for the large bird feeders because they can be transferred from the second milling station to the first but not vice versa. The disruption of Shuttle A has a similar effect in Phase II as in Phase I. The AER drops by 50% and the LER falls to an even smaller value to reflect that an even greater percentage of the production paths have been eliminated as a result of the disruption. That said, the addition of the redundant milling station has dramatically improved the AER and LER values in response to a broken tool. The zero AER and LER values in response to a fault in the lamination station highlight the production system's vulnerability. Collectively, these results show that adding redundancy must be done judiciously. In this case, tools may break often and so redundancy there may be extremely valuable. It's also worth noting that an additional milling station adds three new structural degrees of freedom whereas an additional lamination station would only add one. The proposed AER and LER measures thus have the potential to objectively inform investment decisions in how to upgrade the LFES.

In Phase III, all of the system's resources are redundant – but not for all products. The large bird feeders are still entirely disrupted by the failure of Shuttle A and this is similarly reflected in the AER and LER values. Because, the structural degree of freedom approach differentiates system resources in terms of their processes and their applicability to services, system vulnerabilities can be more easily resolved. Interestingly, the need for resilience in production systems *may* go counterflow to lean manufacturing trends where non-value adding processes were systematically eliminated. In this case, the lack of redundancy in the transportation system eliminates the utility of the other forms of redundancy for half of the product line. For the other two disruptions, the results are predictable and intuitive. The AER measure provides a value of 1 to show that all the products can continue to be produced feasibly, while the LER measure gives a value of 0.5 to reflect the corresponding loss in the percentage of production paths.

VI. Discussion

This section discusses some of the advantages of the proposed resilience measures relative to the existing literature.

The measurement of resilience as an output in response to a disruption as an input is a natural first step. However, when complex simulation packages are used for this purpose [5], [14], [16], [18], [19], [53], they effectively become "black-boxes" that do not necessary shed light as to *why* a particular disruption leads to a particular change in performance. The predictive capability of such black-box measurement is particularly uncertain for large complex systems and often requires exhaustive simulation. In contrast, the proposed resilience measures provide a measurement method based upon abstracted physical models which provide an advisory capability on *how* to best improve the system resilience. That is not to say that these measures inform why disruptions occur. Rather, they inform why a particular disruption affects the system performance more than another.

The proposed resilience measures also present a useful level of abstraction in successive iterations of design and planning. Focusing on the specifics of a system's dynamic equations of motion in a first pass analysis, especially when the full constitutive relations may not yet be fully known, masks the strengths and weaknesses of the underlying structure. If necessary at a later stage, the engineering performance measure in Equation 30 can be detailed such that $Q_i(\psi_1, \psi_2)$ depends explicitly on the structural degrees of freedom. Such an approach is consonant with conventional industrial practice of N-1 contingency analysis [54] in power grids whose results often depend more on the sparsity of the bus admittance matrix than the values of the impedances and bus injections. Similarly, this approach can be extended into a discrete-event system simulation but provides analytical capability at a higher level of abstraction prior to the implementation of a full simulation.

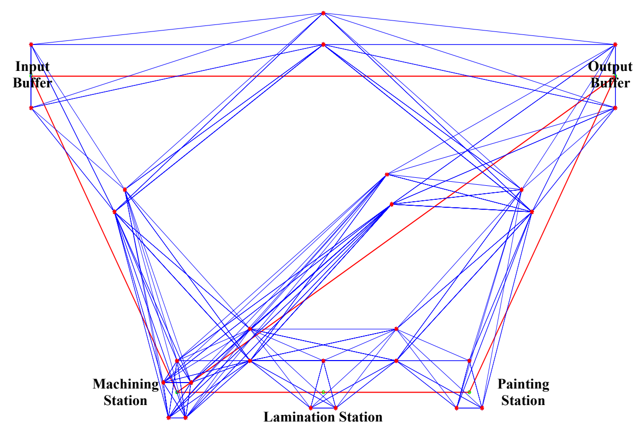


Fig. 6. Traditional (red) & Structural Degree of Freedom-based (blue) Graphs for Starling Manufacturing System in Phase I.

While the proposed measures incorporate many of the concepts found in the existing graph theoretic resilience measurement literature, the most fundamental difference is that the graphs presented in this work have nodes defined as structural degrees of freedom rather than simply locations. Figure 6 highlights the differences between the two approaches with a traditional graph colored in red while the structural degree of freedom based graph colored in blue. Naturally, the values of graph theoretic measures like centrality would differ between the two graphs. The new approach allows for the explicit description of multiple modes of transport (i.e. demonstrated by the nearly parallel lines between buffers) and resources with multiple transformation process (i.e. demonstrated by clusters around each buffer). In that regard, Axiomatic Design [20] and SysML [42] have the potential to bridge the gap from abstract graph theoretic descriptions to more well-established engineering practice. Indeed, the application of model-based systems engineering can allow for the axiomatic design knowledge bases to be developed or mined automatically even for very large scale systems. In such an instance, sparse matrix methods can be applied to take advantage of the sparsity of the knowledge bases and constraint matrices [25], [55]. It is important to note that the graph theory community already has well established case studies for graphs on the order of 10^7 nodes [56]. That said, the application of the axiomatic design model at multiple levels of functional and physical abstraction is useful to tackle problems of particularly large scale.

That said, the proposed resilience measures share many concepts found in the existing graph theoretic resilience measurement literature. First, as demonstrated by the illustrative example, the measures proposed here on the basis of service-specific paths are consonant with resilience measures based upon degree and eigenvector centrality [49], [52]. Service-specific paths are chosen over centrality because they may be more easily linked to physical values that provide different levels of value. Second, the use of paths in the proposed measures also agree with resilience measures based upon connectedness [11], [17], [57]–[61]. While such measures are important in transportation and communication networks where every location must be linked, many resilience improvement strategies *neither require nor encourage* graph connectedness. In power grids, for example, the recent literature encourages the design and operation of multiple microgrids which can connect and disconnect from each other while each serving their local demand for power [62], [63]. More fundamentally, electrical load can be served with onsite generation and without a power grid at all. Similarly, many regions do not require water grids because of the existence of local wells and lakes. Also similarly, production systems have long been designed and operated as manufacturing cells with either static or dynamic configurations. The proposed work is also similar to resilience measures based upon shortest path length [49]–[52]. The dependence on the number rather than the length of the paths takes into consideration LFESs where the valuable artifact travels exceptional fast. For example, angular stability oscillations caused by an outage in Florida could be felt in Minnesota in less than three seconds [64]. Similarly, fiber optic networks rely on information transmission at the speed of light.

VII. Conclusion

This paper has proposed actual and latent engineering resilience measures for large flexible engineering systems. This development was founded upon graph theory, and axiomatic design for large flexible engineering systems. Central to the development was the concept of structural degrees of freedom as the available combinations of systems processes and resources which could be measured individually to describe system capabilities or measured sequentially to give a sense of the skeleton of a system's behavior. It was these sequence-dependent degrees of freedom that were used to enumerate service paths through a LFES. Each service path may be viewed as a value chain through an LFES along which quantities of valuable artifacts flow. Therefore, this work compared the value and quantity of service paths before and after a disruption as measures of static resilience – or survivability.

The work presented in this paper leaves open many opportunities for future work. First, this work can be directly applied to *hetero-functional* LFES's such as tthe energy-water nexus [32], [65], [66], and the transportation electrification [25], [28]. Second, some faults may not be binary and may lead to impaired rather than fully disabled functionality. Third, this work can be extended to address the dynamic aspects of resilience. To that effect, the recent work on reconfiguration processes [48] may be applied as a promising avenue for future developments. Furthermore, the Axiomatic Design knowledge base and the associated structure degree of freedom of measures have been recently used in the design of multi-agent systems in power systems [29], [30], [62], [63] and the manufacturing systems domain [31]. Finally, the authors believe that this work has direct application to resilient control systems and resilient human operation in LFESs that possess a significant amount of control, automation, and intelligence.

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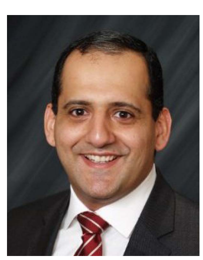

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