Abstract

For many decades, it has also been a critical technology for energy management systems utilized by power system operators. Over time, it has become a mature technology that provides an accurate representation of system state under fairly stable and well understood system operation. The integration of variable energy resources such as wind and solar generation, however, introduce new dynamics and uncertainties into the system. Along with an increase in variability which needs real time monitoring, state estimation (SE) will be extended to the distribution networks which expands the size of the problem. Conventional solutions to this problem result in large problem sets which need to be solved at a faster rate thereby dramatically increasing the computational intensity. This work builds upon two recent SE methods which incorporate event-triggering such that the state estimator is only called in the case of considerable novelty in the evolution of the system state. The first method incorporates only event-triggering while the second adds the concept of tracking. Both SE methods are demonstrated on the standard IEEE 14-bus system and the results are observed for a specific bus for two difference scenarios: (1) a spike in the wind power injection and (2) ramp events with higher variability. Relative to traditional state estimation, the numerical case studies showed that the proposed methods can result in computational time reductions of 90%. These results were support by theoretical discussion of the computational complexity of three SE techniques. The work concludes that the proposed SE techniques demonstrate practical improvements to the computational complexity of classical state estimation. In such a way, state estimation can continue to support the necessary control actions to mitigate the imbalances resulting from the uncertainties in renewables.

Keywords: State Estimation, Event triggered, Tracking, Smart Grid, Renewable Energy Resources, Variable Energy Resources, Computational complexity

1. Introduction

State estimation is an essential method in control system engineering where the state of the system needs to be ascertained from potentially uncertain measurements of a partially understood dynamic system. It has been applied to many industrial applications including motors [1], robots [2], as well as bio- and chemical processing [3, 4]. For many decades, it has also been a critical technology for energy management systems utilized by power system operators [5]. Over time, it has become a mature technology that provides an accurate representation of system state under fairly stable and well understood system operation. Because of the geographical distribution of the power system, it’s state can not be observed directly. Instead, it must be inferred from measurements that include active power injections, reactive power injections, active power flow, reactive power flow, voltage magnitude and phase angle [6]. Although, these measurements (z) may contain errors or noise, the value of the state estimator is in its ability to give the least square error estimate of voltage magnitudes (V) and phase angles (θ) at every bus in a given power grid. This “best” estimate of system state is essential for power system operators to formulate appropriate downstream control actions.

In recent years, the growing demand for energy has resulted in the expansion of the power generation portfolio to include renewables such as solar and wind power. These Variable Energy Resource (VERs) inject uncertain amounts of power at time scales faster and generally dissimilar to that previously found in typical load profiles due to the unpredictable weather conditions [7]. Figure 1 shows the power spectra of power spectra of real power output from two wind farms and one solar PV array [8]. As shown in Figure 2 [9], these power spectra have significantly different spectral content than that found in load spectra [10, 11]. Furthermore, our current abilities to forecast VER power output is also significantly less mature [12]. The resulting forecast errors are a reliability risk that power system operators must actively manage. Fundamentally, the dynamic nature of the power grid requires improvements in monitoring techniques to enhance the downstream situational awareness and decision making [13]. One high priority in that regard are the power grid buses at which VERs are sighted.

The emergence of variable energy resources also unhinges many of the conventional assumptions upon which the power grid was built. Traditionally, power networks consist of 1.) a meshed transmission network connect-
ing centralized generation units in a wide area, and 2.) a radial distribution networks delivering power to the final consumer. The former was viewed as more dynamic and requiring active monitoring and control. The latter was often treated fairly passively. This clear distinction between the transmission and distribution networks allows the study of the two types of networks separately and encouraged different standards and requirements for each type of network[14]. State estimation, as a classical technology of the transmission system, was designed to pick up bulk load variations in relation to their potential impacts on large scale centralized generation units. VERs, however, do not typically have the same technical and economic scale and are often sighted within the distribution system. As a result, the associated uncertain and nondispatchable dynamics require the scope of monitoring to be extended to include the distribution system.

Extending the traditional deployment of state estimation in transmission systems’ energy management systems towards the distribution system dramatically increases the computational load. Figure 3 shows a network graph of the Western Electric Grid in the United States[15]. It shows a highly meshed transmission network connecting the highly radial distribution network. The number of nodes in the former is relatively small as compared to the latter. Although SE methods might be included in Distribution Management Systems (DMS)[16], such a strategy would result in a dramatic increase in the number of buses (or nodes) per unit area. The resulting computational expense restricts the ability to sample at higher speeds to improve monitoring. This paper attempts to address the need for real-time monitoring of power systems with integrated variable energy resources while taking into account the practical implications of increased network size, higher frequency variability, and limited computational capability.

A traditional implementation of state estimation is one that is performed at fixed intervals. This may not lead to an accurate representation of state for a power system with integrated VERs. The Classical State Estimation (CSE) uses a Weighted Least Square (WLS) algorithm to estimate the state vector. The algorithm is performed at regular intervals to update the state vector $x = [V, \theta]$. For the reasons stated above, the fixed interval CSE is unlikely to be able to continue to provide high fidelity in the value of the state variables[17, 18].

While many developments in the academic literature are adding computational complexity to state estimation techniques [19–23], this paper looks to event triggering techniques to reduce it. In [17], the concept of Event Triggered State Estimation (ETSE) using the variability in the wind was introduced. It proposes to perform the state estimation only when triggered by considerable “novelty”
in field measurements such as significant changes in the nodal power injections from wind power. In a complementary work[18], the concept of tracking state estimation [24] was combined with event-triggering. This paper compares these two methods from a numerical as well as a theoretical perspective.

The remainder of the paper develops into six sections. Section 2 provides background material on classical state estimation and variable energy resources. Section 3 then develops the methodology of the two event-triggered state estimation techniques. Section 4 follows with a description of how these methods were implemented in simulation. The results are combined with an extended discussion in Section 5. The paper is brought to a conclusion in Section 6.

2. Background

This section provides the background material on classical state estimation and variable energy resources required for the developments in the later sections.

2.1. Classical State Estimation and Weighted Least Square (WLS)

CSE was introduced to power grids in 1971 by Fred Schweppe [5] and since then has been an area of extensive research. There are several algorithms which have been developed to perform the estimation[23][25]. The WLS algorithm is widely used and is adopted in this paper.

Given an N bus network, the state vector at time k is \( x(k) = [\theta_1 \ldots \theta_N, V_1 \ldots V_N]^T \) where \( \theta_i \) and \( V_i \) are the voltage phase angle and magnitude respectively at the \( i^{th} \) bus. The state vector \( x(k) \) is derived from a measurement set \( z(k) \) of length \( M \) which is obtained from the network through the supervisory control and data acquisition (SCADA) system [26]. As mentioned in Section 1, the measurements include the active power injection \( P_i \) and reactive power injection \( Q_i \) at a bus \( i \), the active power flow \( P_{ij} \) and the reactive power flow \( Q_{ij} \) between buses \( i \) and \( j \), voltage measurements \( V_i \) or time stamped measurements from the phasor measurement units (PMUs) \( (V_i, \theta_i) \) [27].

The measurement set must include a set of independent measurements of size greater than the length of the state vector i.e. \( M \geq 2N \) [28]. This ensures that the system is completely observable. Complete observability ensures that the measurements are sufficient to arrive at accurate estimates for all the state variables.

The measurement vector \( z(k) \) is related to the state vector \( x(k) \) through [5]:

\[
z(k) = h(x(k)) + \epsilon(k)
\]  

where \( h(x(k)) \) is the function vector of length \( M \) that consists of the power flow equations that define power injections into buses and flows within branches.Explicitly, they are[29]:

\[
P_i = |V_i| \sum_{j=1}^n |V_j|(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)) \tag{2}
\]

\[
Q_i = |V_i| \sum_{j=1}^n |V_j|(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)) \tag{3}
\]

\[
P_{ij} = |V_i||V_j|(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)) - G_{ij}|V_i|^2 \tag{4}
\]

\[
Q_{ij} = |V_i||V_j|(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)) + B_{ij}|V_i|^2 \tag{5}
\]

where \( G_{ij}, B_{ij}, \theta_{ij} \) are the conductance, the susceptance and the voltage angle difference between buses \( i \) and \( j \) from \( h(x) \), a \( M \times 2N \) Jacobian matrix, \( H \), can be defined

\[
H(x) = \frac{\partial h(x)}{\partial x} \tag{6}
\]

The measurement error \( \epsilon(k) \) at time \( k \) is assumed to be normally distributed such that a weighting matrix, \( W \), can be constructed for the individual measurement error variances \( \sigma^2 \) as

\[
W = \text{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_M^2) \tag{7}
\]

The WLS formulation of CSE is then presented as a minimization of the square error [28]:

\[
\text{Minimize } f = [z(k) - h(\hat{x}(k))]^T W [z(k) - h(\hat{x}(k))] \tag{8}
\]

s.t. \( z_i(k) = h_i(x(k)) + \epsilon_i(k), \ i = 1 \ldots M \tag{9} \)

An iterative Newton-Raphson procedure is used to solve for the state estimate \( \hat{x}(k) \) by normal equations as presented in Algorithm 1 [6]. In CSE, the Algorithm 1 is performed at regular intervals to update the state vector.

**Algorithm 1**

1. Receive \( z \) from the SCADA system
2. Initialize the state vector \( x = x^c \) and the iteration counter \( c \)
3. Compute the measurement residual

\[
r^c = z - h(x^c)
\]

4. Obtain \( H(x^c) \) and \( G(x^c) \)

\[
H(x^c) = \frac{\partial h(x^c)}{\partial x} \quad \text{and} \quad G(x^c) = H^T(x^c)W H(x^c)
\]

5. Solve for the linear system

\[
G(x^c) \Delta x^c = [z - h(x^c)]^T W [z - h(x^c)]
\]

6. Update the state vector and the iteration counter

\[
x^{c+1} = x^c + \Delta x^c, \quad c = c + 1
\]

7. Check convergence criterion at a maximum count \( c_{max} \)
8. If convergence is satisfied:

\[
\hat{x} = x^c
\]

9. If not, return to Step 1
2.2. Variable Energy Resources

The variability characteristic of solar and wind generation as they are integrated into the power grid necessitates the need for improved real-time monitoring which subsequently improves situational awareness, decision-making and automatic control [30]. These sources are called variable because of two complimentary characteristics: uncertainty and intermittency. They are uncertain in that their inputs of solar irradiance and wind speed are stochastic in nature and hence require prediction. Forecast model accuracy for wind or solar energy has improved in recent years and remains as a field of active research [12, 31, 32]. VERs are intermittent in the sense that they are not dispatchable like conventional generators and hence introduce a new set of dynamics into the power grid.

In order to incorporate VERs into an ETSE, a stochastic input-to-output model of the VER is required. Solar photovoltaic models are essentially Norton equivalents with a shunt diode. They differ in their inclusion of the shunt and Norton resistance. These models can be solved to relate the solar irradiance to the maximum power output [29, 33]. An even greater diversity of models exist for wind turbines [34–36]. Furthermore, the in-built controllers can reduce the variability in the wind power output, but wind speed ramps or sudden gusts may still cause significant fluctuations.

For the purposes of the developments later in this paper, a simple model relating the wind power output $P_e$ to the wind velocity $v$ is described [37].

$$P_w(v) = \frac{1}{2} \rho A v^3. \quad (10)$$

where $P_w$ is the wind power, $\rho$ is the air density and $A$ is the area of cross section of the flow tube. The wind power is related to the mechanical power $P_m$ from the turbine through [37]

$$P_m = C_p \times P_w(v) \quad (11)$$

The turbine coefficient, $C_p$, varies with the turbine design [37] and could be modeled as non-linear algebraic equation [29]. The electrical power output is proportional to the mechanical power of the turbine through [37]

$$P_e = \eta \times P_m \quad (12)$$

where $\eta$ is the generator efficiency and it varies with the choice of generator [38].

The wind active power injection is then ultimately tied to the state vector via Equation 2. The effect of wind generation on the phase angle has been establish by Allen in Texas-EROCT synchrophasor network [7].

3. Methodology

In this section, two event triggered state estimation algorithms are introduced; one without and another with tracking.

Traditionally, CSE executes the WLS algorithm at regular but relatively slow intervals (10-30s) under the assumption that the power system under observation evolves quasi statically between consecutive executions of the WLS. The increasing penetration of VER in recent years has introduced greater dynamics into the power system thus potentially violating this assumption. To keep up with the variation in the states, reductions in the CSE execution interval has been proposed as a solution. Here, the concept of event-triggering is introduced in Section 3.1 to give a more computational efficient state estimation solution. Section 3.2 enhances this method further with the concept of tracking.

3.1. Event Triggered State Estimation (ETSE)

The concept of event triggered state estimation is to update the state vector only when there is considerable novelty in the measurement set [39]. In [39] the trigger is set by the square error of the current measurement and the previous estimate. In [17], the trigger is set based on the novelty in the power injections at the bus with VER generation. The event is identified using the Western Electric Rules (WER) [40] often used in stochastic process control.

The WER have been formulated to pick up non-random variations or trends in the process output so that a necessary control action can be implemented. In [17], the rules are applied to observe the active power injection at the buses with wind generation over a window that grows in size until the conditions of the triggering criterion are met; at which point it resets back to a size of two.

The rules are defined with respect to a central limit and the distance of recent measurements from the central limit. The distance is defined by the standard deviation of the measurements in a given observation window. The central limit $\mu$ is taken as the average of the measurement over a period of time or the expected value of the measurement. The WER are [40]:

- If the measurement point lies outside $\mu \pm 3\sigma$,
- If two out of three measurement points are outside $\mu \pm 2\sigma$,
- If four out of five measurement points are outside $\mu \pm 1\sigma$,
- If eight consecutive points are on either side of $\mu$

Formally, the application of event triggering is as follows. Let the active wind power injection at time $k$ at bus $i$ be $P_{w,i}(k)$ and $\mu = \bar{P}_{w,i}(k)$ be the average of the measurements within a given observation window of size $S_t(k)$:

$$S_t(k) = \begin{cases} 2; & \tau(k-1) = 1 \\ S_t(k-1) + 1; & \tau(k-1) = 0 \end{cases} \quad (13)$$
\[ P_{w,i}(k) = \sum_{j=0}^{S_i(k)} P_{w,i}(k-j) \frac{S_i(k)}{S_i(k)} \]  

(14)

where \( \tau(k) \) is a Boolean trigger which decides whether to perform state estimation upon receiving \( P_{w,i}(k) \). The value of the previous trigger is used to update the window size.

The Western Electric Rules (WER) are able to pick up events such as ramps but cannot recognize random events such as gusts which may result in sudden spikes. In order to capture such an event, an additional criterion is introduced as follows:

\[ |P_{w,i}(k) - P_{w,i}(k-1)| > \alpha \times P_{w,i}(k-1) \]  

(15)

where \( \alpha \) is a tuning parameter that the implementer can choose to differentiate between strong gusts and noise. Together, this criterion and WER are combined to explicitly state the full triggering function \( \tau(k) \),

\[
\tau(k) = \begin{cases} 
1, & P_{w,i}(k) \geq (P_{w,i}(k) + 3\sigma_i) \\
1, & P_{w,i}(k) \leq (P_{w,i}(k) - 3\sigma_i) \\
1, & \sqrt{2} \leq (P_{w,i}(k-j) \geq (P_{w,i}(k) + 2\sigma_i)) \geq 2 \\
1, & \sqrt{2} \leq (P_{w,i}(k-j) \leq (P_{w,i}(k) - 2\sigma_i)) \geq 2 \\
1, & \sqrt{4} \leq (P_{w,i}(k-j) \geq (P_{w,i}(k) + \sigma_i)) \geq 4 \\
1, & \sqrt{4} \leq (P_{w,i}(k-j) \leq (P_{w,i}(k) - \sigma_i)) \geq 4 \\
1, & \sqrt{4} \leq (P_{w,i}(k-j) \geq P_{w,i}(k)) \geq 8 \\
1, & \sqrt{4} \leq (P_{w,i}(k-j) \leq P_{w,i}(k)) \geq 8 \\
1, & |P_{w,i}(k) - P_{w,i}(k-1)| > \alpha \times P_{w,i}(k-1) \\
0, & Otherwise 
\end{cases}
\]

(16)

### 3.2. Event Triggered Tracking State Estimator (ETTSE)

In addition to the classical state estimator, Schweppe also proposed the concept of a tracking state estimator to update the successive state estimates \( \hat{x}(k) \)[24]. This method utilizes the available state estimate \( \hat{x}(k) \) and the current measurement set \( z(k) \) to evaluate the update \( \Delta x(k) \) which should be added to the state estimate \( \hat{x}(k-1) \) to obtain \( \hat{x}(k) \)[24].

\[
\hat{x}(k) = \hat{x}(k-1) + \Delta x(k) 
\]

(17)

The \( \Delta x(k) \) is obtained by solving the following[24]:

\[
\Delta x(k) = G^{-1}(\hat{x}_t)H(\hat{x}_t)(W[z_k - h(\hat{x}(k-1))]) 
\]

(18)

where \( \hat{x}_t \) is the state vector obtained from the last time Algorithm 1 was executed.

The inverse matrix, \( G^{-1}(\hat{x}_t) \) (also known as the gain matrix) and the Jacobian, \( H(\hat{x}_t) \) are calculated only when the trigger is set. The gain matrix and the Jacobian matrix once calculated retain their value till the next triggered event is identified. During the period between consecutive events the state variable is updated by Equation (17). The method of tracking is computationally less intensive than the optimization program outlined in Algorithm 1 because the Gain matrix is not computed for each update.

The tracking state estimator is then combined with the event-triggering to further reduce the computational demand[18]. The inverse matrix, \( G^{-1}(\hat{x}_t) \) (also known as the gain matrix) and the Jacobian, \( H(\hat{x}_t) \) are calculated only when the trigger is set. The gain matrix and the Jacobian matrix once calculated retain their value till the next triggered event is identified. During the period between consecutive events the state variable is updated by Equations 17 and 17. The method of tracking is computationally less intensive than the optimization program outlined in Algorithm 1 because the Gain matrix is not computed for each update. This enhanced method is described in Algorithm 2.

**Algorithm 2**

1. Receive active power measurement \( P_{w,i}(k) \) from all the buses with wind generation
2. Update \( S_i(k) \) and compute \( \mu = P_{w,i}(k) \) using equation (13), (14) respectively.
3. Update \( \tau(k) \) using Equation (16)
4. If \( \tau(k) = 1 \), receive measurement set \( z \) of the entire network and perform SE using Algorithm 1 to update \( \hat{x}(k) \)
5. If \( \tau(k) = 0 \) receive measurement set \( z \) of the entire network and calculate \( \Delta x(k) \) using Equation 18 and \( \hat{x}(k) \) using Equation (17).
6. Wait for active power measurement \( P_{w,i}(k) \)

![Figure 4: Event triggered tracking state estimation algorithm](image-url)
4. Simulation Implementation

The standard IEEE 14 bus system is used to illustrate the two event-triggered state estimation techniques developed in this paper. The measurement set is predefined to be: Voltage measurement at bus 1; power injection (active and reactive) at buses 2, 3, 7, 8, 10, 11, 12 and 14 and active and reactive power flow on the branches between the following bus pairs: 1-2; 2-3; 2-4; 2-5; 4-5; 4-7; 4-9; 5-6; 6-11; 6-13; 7-9 and 12-13.

Figure 5: IEEE 14 Bus system[29]

Standard methods of observability [28] are applied to assure that the measurement set results in a full column rank of H given by Equation (6). The tuning parameter in Equation (15) is set to 0.25.

The following 3 scenarios are observed and compared against each other:

- Execute Algorithm 1 every 2s interval which is referred to as Fast SE (FSE)
- Perform Event Triggered State Estimation (ETSE) [17]
- Perform Event triggered Tracking State Estimator (ETTSE) using Algorithm 2

The FSE is considered an ideal scenario which is used as the benchmark to assess the ETSE and ETTSE. Two different wind profiles are used to test the three scenarios. In each case, the wind speed follows the composite wind speed model presented in [29]. This is taken as a stochastic input to the DFIG wind turbine model provided in Sim-Power [41] to obtain the variability in power injection at Bus 2. These results are integrated into the IEEE 14-bus system. MATPOWER[42] is used to obtain the power flow measurements for variation in the power injection.

A normally distributed measurement error of standard deviation 0.07 per unit (p.u) and mean 0.06 p.u is introduced.

The entire simulation is conducted within a Matlab environment on a Windows 7 HP laptop with an Intel Core i5 CPU running at 2.27Ghz.

The results of the simulations are assessed on the basis of the relative error and computation time. Relative error is assessed on the basis of the norm

\[
\text{norm}(err) = \sqrt{\sum_{j=1}^{n} (err_j)^2}
\]

5. Results and Discussions

The results in the variation of the phase angle at Bus 5 for two different wind profiles are studied. Case 1 represents a spike in the wind power injection while Case 2 represents ramp events and high variability. Also, the \( \Delta x \) is observed and compared to the difference between the real update obtained from the FSE. The relative error is used to compare the fidelity of the ETSE and ETTSE approaches.

Table 1 shows the relative error for the phase angle at Bus 5 for the ETSE and the ETTSE. In both cases of wind power injection profile, the ETTSE has significantly less error than the ETSE. The ETTSE error is approximately 95% smaller than the ETSE for both cases. To illustrate the comparison more dramatically, reductions in the error resulting from each algorithm is given in Figure (6) for each case.

<table>
<thead>
<tr>
<th>Case</th>
<th>ETSE</th>
<th>ETTSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>(1.04 \times 10^{-2})</td>
<td>(3.94 \times 10^{-4})</td>
</tr>
<tr>
<td>Case2</td>
<td>(2.9 \times 10^{-2})</td>
<td>(1.2 \times 10^{-3})</td>
</tr>
</tbody>
</table>

Table 1: Relative error of the two SE approaches compared to FSE

Figure 7 and Figure 8 indicate the variation of phase angle at Bus 5 measured by each SE approach for Case 1. By comparing the two time series, the ETTSE demonstrates that it can capture the modulations in phase angle for different wind profiles more faithfully. Rather than
leaving the state estimate fixed between triggers, the linear approximation of $\Delta x(k)$ allows the state estimate to be update between successive event triggers of Algorithm 1 and so limit the evolution of errors at each time step.

The accuracy of the update $\Delta x$ from the ETTSE can be compared directly to the difference $\hat{x}(k) - \hat{x}(k-1)$ from the FSE. Figure 9 shows the update of the phase angle estimate at Bus 5 in Case 1. The time series from the two methods closely follow each other with the ETTSE doing so in much less computation time.

The relative merits of the three state estimation techniques can also be compared from the perspective of computation time. Figure 10 shows the computation time for three SE techniques for both cases. In both numerical cases, the introduction of event-triggering lead to 4-5x reduction in the required computation. To that effect the ETSE and ETTSE were quite similar to within a few percent. The incremental increase in computation of the ETTSE, however, is worth the large improvement in accuracy previously shown in Figure 6. One can conclude that the calculation of the update from Equations 17 and 18 are computationally light to warrant its application at every time step.

The computational savings associated with the numerical experiments of Cases 1 and 2 can be more deeply addressed by comparing the computational complexity\cite{43,44} of the respective state estimation techniques. To that effect, two assumptions are made:

1. Arithmetic operations of individual floating-point elements has a computational complexity value of $O(1)$
2. $h(x)$ and $W$ are already available in memory prior to start.

It follows that from Algorithm 1 that the computational complexity of the classical state estimator $C_{CSE}$ is:

$$C_{CSE} = [O(M_I(N)) + 5 O(M_M(N)) + 5 O((N))]N_c$$

(20)

where $M_I(N)$ and $M_M(N)$ are the computational complexity of matrix inversion and multiplication and $N_c$ is the average number of iterations required for convergence of
Algorithm 1. Using the best available numerical methods, $M_I(N) = \mathcal{O}(N^{2.373})$ and $M_{II}(N) = \mathcal{O}(N^{2.373})[43, 44]$. Therefore, the highest order term of the computational complexity becomes:

$$C_{CSE} = \mathcal{O}(6N_c \times N^{2.373})$$  \hspace{1cm} (21)

Now consider a time period $T$ in which state estimation is to occur with a time resolution of $\delta t$. The computational complexity over the full period is

$$C_{CSE_T} = \mathcal{O}\left((6N_c)\left(\frac{T}{\delta t}\right)N^{2.373}\right)$$  \hspace{1cm} (22)

In contrast, consider an event triggered state estimation algorithm which has an average event frequency of $1/(\delta T)$ seconds. The computational complexity over the full period is

$$C_{ETSE_T} = \mathcal{O}\left((6N_c)\left(\frac{T}{\delta T}\right)N^{2.373}\right)$$  \hspace{1cm} (23)

The event triggered tracking state estimation algorithm adds the computation of the update associated with Equation 18 at every time step. Its computational complexity over the full period is

$$C_{ETTSE_T} = \mathcal{O}\left((6N_c)\left(\frac{T}{\delta T}\right)N^{2.373} + 3\left(\frac{T}{\delta T} - 1\right)N^{2.373}\right)$$  \hspace{1cm} (24)

From Equations 22-24, a number of conclusions can be reached on the speed up of the ETSE, ETTSE techniques relative classical state estimation. First, in all three equations, the computational complexities are dominated by the polynomial order dependence on the number of buses. As the scope of state estimation expands to include the distribution system, the number of buses can be seen to expand exponentially. Computationally, this exponential growth would be compounded by the $2.373$ exponent of matrix inversion and multiplication. The trends to decrease $\delta t$ with the introduction of VERs implies an additional asymptotic growth of computational complexity. The ETSE algorithm relieves some of this burden by assuming that $\delta T \gg \delta t$; their ratio being the average speed up. Finally, the ETTSE algorithm recognizes that the Newton-Raphson approach described in Algorithm 1 may many require iterations in order to converge. $N_c >> 1$. Therefore, the computational complexity associated with the tracking update shown in the second term of Equation 18 can be viewed as incremental in comparison to the first term. In all, the ETSE and more so the ETTSE can be viewed as practical improvements to CSE in light of trends associated with VER integration.

6. Conclusion

The integration of VERs into the power grid stresses the need for improved state estimation techniques that can keep up with the required computational requirements. This paper contrasted two SE techniques which incorporated event-triggering and state tracking to meet the real-time monitoring demands of a dynamic power grid with reduced computational time. The ETSE algorithm proposed in this paper uses a trigger criterion based on the power output of the VERs in the network. The trigger criterion includes the historical data of the power output and uses the VER from stochastic process control to identify events such as ramps and surges in the power output. The new update mechanism is used in the ETTSE to track variables between two consecutive triggers. Consequently, the numerical case studies demonstrated a 95% reduction in errors in the ETTSE relative to the ETSE. The ETSE and ETTSE were also shown to theoretically reduce the computational complexity of CSE. This was further supported by numerical case study which showed a 90% improvement relative to the FSE.

While the ETTSE does show significant advantages over CSE, the authors recognize the need for further work. Ultimately speaking, the highest order computational complexity term is the $2.373$ associated with the number of buses in the system. Therefore, the greatest improvements can be made by developing techniques that take advantage of the natural sparsity and distribution of the power grid. In that regard, ongoing developments in distributed and hierarchical state estimation techniques appear promising[19–23]. Distributed state estimation[4] techniques fundamentally convert the computational complexity problem from $N^{2.373} \rightarrow N_A (N/N_A)^{2.373}$ where $N_A$ is the number of areas into which the problem is decomposed. Similarly, hierarchical SE methods present some promise if the lower level(s) of SE can take advantage of the geographical density of information. Finally, sparse matrix multiplication methods can take advantage of the natural sparsity in the power flow equations and the weighting matrix.

References


