A Hybrid Dynamic System Model for Multi-Modal Transportation Electrification

Amro M. Farid, Senior Member, IEEE

Abstract—In recent years, transportation electrification has emerged as a trend to support energy efficiency and CO_2 emissions reduction targets. The true success, however, of this trend depends on the successful integration of electric transportation modes into the infrastructure systems that support them. Left unmanaged, plugin electric vehicles may suffer from delays due to charging or cause destabilizing charging loads on the electrical grid. Online electric vehicles have emerged to remediate the need for stationary charging and its effects. While many works have sought to mitigate these effects with advanced control functionality such as coordinated charging, vehicle-to-grid stabilization, and charging queue management, few works have assessed these impacts as a holistic transportation-electricity nexus. This paper develops a hybrid dynamic system model for transportation electrification. It also includes “next generation” traffic simulation concepts of multi-modality and multi-agency. Such a model can be used by electrified transportation fleet operators to not just assess but also improve their operations & control. The hybrid dynamic system model is composed of a marked Petri-net model superimposed on the continuous time kinematic and electrical state evolution. The model is demonstrated on an illustrative example of moderate size and functional heterogeneity.

Index Terms—transportation electrification, multi-modal transportation, power systems, hetero-functional networks, hybrid dynamic system, Petri nets, Axiomatic Design

I. INTRODUCTION

In recent years, electrified transportation has emerged as a trend to support energy efficiency and CO_2 emissions reduction targets [1]–[5]. Relative to their internal combustion vehicle (ICV) counterparts, electric vehicles (EV), be they trains, buses, or cars, have a greater “well-to-wheel” energy efficiency [5], [6]. They also have the added benefit of not emitting any carbon dioxide in operation and rather shift their emissions to the existing local fleet of power generation technology [7].

The true success of electric vehicles depends on their successful integration with the infrastructure systems that support them. From a transportation perspective, plugin electric cars may only have a short range of 150km [8] but may still require several hours to charge [9]. This affects when a vehicle can begin its journey and the route it intends to take. From an electricity perspective, the charging loads can draw large power demands which may exceed transformer ratings, cause undesirable line congestion, or voltage deviations [10]–[13]. These loads may be further exacerbated temporally by similar charging patterns driven by similar work and travel lifestyles or geographically by the relative sparsity of charging infrastructure in high demand areas [11]. In effect, the electric vehicles and their supporting charging infrastructure couples the transportation and electrical systems into a nexus.

Definition 1. Transportation-Electricity Nexus (TEN) [14], [15]: A system-of-systems composed of a system with the artifacts necessary to describe at least one mode of electrified transport united with an interdependent system composed of the artifacts necessary to generate, transmit, distribute and consume electricity.

As a result, the performance in the transportation and electrical domain can not be independently studied. Furthermore, efforts to operate and control the performance in either domain requires an assessment model whose scope includes the functionality of both systems. Consider an EV taxi fleet operator [11]–[13]. They must dispatch their vehicles like any other conventional fleet operator, but with the added constraint that the vehicles are available after the required charging time. Once en route, these vehicles must choose a route subject to the nearby online (wireless) and conventional (plug-in) charging facilities. In real-time, however, much like gas stations, these charging facilities may not be available due to the development of queues. Instead, the EV taxi driver may opt to charge elsewhere. Once a set of EV taxis arrive to a conventional charging station, the EV taxi fleet operator may wish to implement a coordinated charging scheme [16]–[27] so as to limit the charging loads on the electrical grid. The local electric utility may even incentivize this EV taxi operator to implement a “vehicle-to-grid” scheme [28]–[30] to stabilize variability in grid conditions. These five transportation-electric nexus operations management decisions are summarized in Table I [15]. While these decisions are coupled, the degree to which they can be coordinated ultimately depends on the presence of a well designed Intelligent Transportation-Energy System (ITES) [12].

<table>
<thead>
<tr>
<th>Table I: Intelligent Transportation-Energy System Operations Decisions in the Transportation Electricity Nexus [15]</th>
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<tbody>
<tr>
<td><strong>Vehicle Dispatch</strong>: When a given EV should undertake a trip (from origin to destination)</td>
</tr>
<tr>
<td><strong>Route Choice</strong>: Which set of roads and intersections it should take along the way</td>
</tr>
<tr>
<td><strong>Charging Station Queue Management</strong>: When &amp; where it should charge in light of real-time development of queues</td>
</tr>
<tr>
<td><strong>Coordinated Charging</strong>: At a given charging station, when the EVs should charge to meet customer departure times and power grid constraints</td>
</tr>
<tr>
<td><strong>Vehicle-2-Grid Stabilization</strong>: Given the dynamics of the power grid, how can the EVs be used as energy storage for stabilization</td>
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</tbody>
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Amro M. Farid is an Associate Professor of Engineering at Thayer School of Engineering at Dartmouth and a research affiliate with the MIT Mechanical Engineering Department amfarid@dartmouth.edu, amfarid@mit.edu
Designing such an intelligent transportation-energy system requires a careful assessment of the couplings between the kinematic and electrical states in a TEN. Thus far, to our knowledge, only three works have studied this coupling. A simplified study based on the city of Berlin has been implemented on MATSIM [30]. Meanwhile, the first full scale study was completed in the city of Abu Dhabi [11]–[13] using the Clean Mobility Simulator [31]. The former assumed a home-charging (i.e., always available) use case and thus neglected the impacts of charging station capacity on the transportation system as well as on the power system. The latter study sought a more holistic approach to system performance measurement. “Quality of Service” (QoS) [11], [13] was introduced as a transportation performance measure to address the availability concerns expressed in EV adoption public attitude surveys. Meanwhile, power system line and bus safety criteria were introduced on the basis of IEEE reliability standards [11]–[13]. A third study focused on the differences between plug-in online electric vehicles [15]. The methodologies for the performance assessment of a TEN are still very much in the course of development.

This paper presents a hybrid dynamic model (HDM) of a transportation electricity nexus. The model uses Axiomatic Design for Large Flexible Engineering Systems to describe the system structure. Timed Petri-nets are then used to track the discrete state of transportation vehicles. Continuous time differential equations are then added in between discrete states to describe the time evolution of the kinematic and electrical states. An earlier version of the model has been reported [14], [15]. The model presented here specifically includes the electrical power grid behavior and adds discussions on multi-modality, multi-agency, and capacity constraints in the electrified transportation system. The development assumes sufficient background in graph theory [32]–[34], timed Petri-nets [35]–[37], microscopic traffic simulation [38]–[40] and power system dynamics [41] which is otherwise obtained from the provided references.

The paper proceeds as follows. Section II provides an introduction to Axiomatic Design for Large Flexible Engineering Systems. Section III then develops the HDM using Axiomatic Design, timed Petri-nets and established continuous-time dynamics of transportation and power grid behavior. Section IV then demonstrates the model on a test case of moderate size. Section VI concludes the work with practical implications on the operations and control of electrified transportation fleet operators.

II. BACKGROUND: AXIOMATIC DESIGN FOR LARGE FLEXIBLE ENGINEERING SYSTEMS

The development of the model in the following section is built upon Axiomatic Design for Large Flexible Engineering Systems (LFES) [42]–[45] rather than graph theory [32]–[34]. This section first motivates the choice of Axiomatic Design, and then proceeds to introduce its fundamental concepts: system processes, system resources, and the allocation of the former to the latter in a knowledge base.

A. Need for Axiomatic Design for LFES

In transportation and power systems, a graph (be it directed or undirected) $D = \{B, E\}$ consisting of nodes $B$ and edges $E$ is often proposed as a modeling and analytical framework. In transportation systems, the nodes often physically represent intersections and stations while edges/arcs represent roads, rails or transportation routes [46]–[50]. In power systems, the nodes often physically represent generators, substations, and loads while the edges represent the power lines [51], [52]. The relationships between these nodes and edges are then analyzed using incidence and adjacency matrices.

While graph theory remains an effective analytical tool, its use of only two types of elements may be ultimately insufficient in syntax for the modeling of hetero-functional systems-of-systems like the transportation-electricity nexus [45]. As a broad consensus in systems engineering, an engineering system requires descriptions of form and function [53]–[56]. The syntax of these descriptions has been formalized into two complementary industrial standards: SysML/UML [55] and OPM [56]. As standards, both of these have been deemed complete in that their syntax describes the breadth of conceptual primitives required by practicing systems engineers. More formally, methods from the ontological sciences have been used to show that UML adheres to the formal definitions of soundness, completeness, laconicity, and lucidity [57], [58]. In contrast, the use of only two types of elements in graph theory prevents it from being lucid and laconic with respect to the breadth of concepts in today’s practice of systems engineering. More intuitively, system’s engineering practice would classify nodes and edges as elements of a system’s form [53]–[56]; neglecting an explicit description of the system’s function [59].

The dual representation switches the roles of nodes and edges [60], [61] but still continues to describe form elements. The use of colored nodes and edges [33], [34] offers the potential for a richer modeling syntax, but the mapping of conceptual primitives to the node and edge coloring is not currently known to our knowledge.

To overcome the limitations described above, Axiomatic Design for LFES Theory is chosen for its ability to explicitly model the allocation of system function to system form as a mathematical equivalent of the SysML graphical representation [42], [45]. Meanwhile, because Axiomatic Design uses binary matrices it lends itself to using many of the analytical techniques found in graph theory. Furthermore, as has been previously described, Axiomatic Design also [14], [15], [45], [62]

1) Explicitly differentiates between heterogeneous functional modes of transport with fundamentally different graphs for each
2) Explicitly describes the system function, especially functions that do not involve a type of transportation.
3) Explicitly describes “null-processes” (i.e. the process of staying in the same location).
4) Facilitates further detailed engineering design in terms of system function and form.
5) Facilitates the description of faulted, or intentionally-offline functionality to support reconfigurable operation.
These features have allowed it to be successfully applied to several relevant large flexible engineering systems including: production systems [43], [44], [63], power systems [64], water distribution systems [45], their nexus [65], [66], transportation systems [14], [62], and transportation electrification [14], [15].

Definition 2. Large Flexible Engineering System (LFES) [42], [45]: an engineering system with many functional requirements that not only evolve over time, but also can be fulfilled by one or more design parameters.

In the context of this work, the functional requirements and design parameters mentioned in Definition 2 are understood to be mutually exclusive and collectively exhaustive sets of the system’s processes (P) and resources (R) respectively. This change of terminology is applied consistently for the rest of the paper.

B. System Resources

Axiomatic Design for Large Flexible Engineering Systems requires the identification of a set of system resources [54] as structural inputs [45]. These resources R = M ∪ B ∪ H may be classified into transforming resources M = {m1, ..., mσ(M)}, independent buffers B = {b1, ..., bσ(B)}, and transporting resources H = {h1, ..., hσ(H)}. The σ() notation gives the size of a set. The set of buffers BS = M ∪ B is also introduced for later simplicity [45]. These resources may be defined at any level of abstraction or decomposition depending on the nature of the application. Nevertheless, it is sometimes useful to aggregate a set of resources R into a set of aggregated resources R̃ with an aggregation matrix A that assigns resource rj to aggregated resource R̃ [43]–[45].

R = A ⊗ R (1)

Definition 3. Aggregation Operator ⊗ [43]–[45]: Given boolean matrix A and sets B and C, C = A ⊗ B is equivalent to:

C(i) = \bigcup_j a(i, j) ∧ b(j) (2)

C. System Processes

Axiomatic Design for Large Flexible Engineering Systems also requires the identification of a set of system processes [54] (defined at any level of abstraction). They are formally classified into three varieties: transformation, transportation and holding.

Definition 4. Transformation Process [43]–[45]: A resource-independent, technology-independent process p\_w = \{p_{w1}, ..., p_{w\sigma(P_w)}\} that transforms an artifact from one form into another.

Definition 5. Transportation Process [43]–[45]: A resource-independent process p\_w = \{p_{w1}, ..., p_{w\sigma(P_w)}\} that transports artifacts from one buffer b_{wy1} to b_{wy2}. There are \sigma^2(B_S) such processes of which \sigma(B_S) are “null” processes where no motion occurs. The following convention of indices is adopted:

u = \sigma(B_S)(y_1 - 1) + y_2 (3)

This convention implies a directed bipartite graph between the set of independent buffers and the transportation processes whose incidence in M_H− and incidence out M_H+ matrices are given by:

M_H− = \sum_{y_1=1}^{\sigma(B)} e_{y_1} \sigma(B) \left[ e_{y_1} \sigma(B) \right]^T (4)

M_H+ = \sum_{y_2=1}^{\sigma(B)} e_{y_2} \sigma(B) \left[ e_{y_2} \sigma(B) \right]^T (5)

where 1^n is a column vector of ones of predefined length n, e_{iy} is the i^{th} elementary basis vector, and \otimes is the Kronecker product.

Definition 6. Holding Process [43]–[45]: A transportation independent process p_{\varphi} \in P_\varphi that holds artifacts during the transportation from one buffer to another.

D. Axiomatic Design for LFES Knowledge Base

Once identified, the system processes are allocated to system resources via the Axiomatic Design equation for LFESs [43]–[45].

P = J_S ⊗ R (6)

where J_S is a binary matrix called a LFES “knowledge base”, and ⊗ is “matrix boolean multiplication”[43]–[45].

Definition 7. LFES Knowledge Base [43]–[45]: A binary matrix J_S of size \sigma(P) × \sigma(R) whose element J_S(w, v) ∈ \{0, 1\} is equal to one when action e_{wv} (in the SysML sense [55]) exists as a system process p_{w} \in P being executed by a resource r_{v} \in R.

In other words, the system knowledge base itself forms a bipartite graph which maps the set of system processes to their resources. J_S can then be reconstructed straightforwardly from smaller knowledge bases that individually address transformation, transportation, and holding processes. P_H = J_M ⊗ M, P_H = J_H ⊗ R, P_H = J_φ ⊗ R. J_S then becomes [43]

J_S = \begin{bmatrix} J_M & 0 \\ 0 & J_H \end{bmatrix} (7)

where in order to account for the simultaneity of holding and transportation processes [63]

J_H = [J_φ ⊗ J^{\sigma(P_\varphi)}] : [1^{\sigma(P_\varphi)} ⊗ J_H] (8)

where · is the Hadamard product.

Interestingly, the LFES knowledge base has an additional property in that it defines the LFES sequence-independent structural degrees of freedom [43]–[45]. These are instrumental in the construction of the state vector in large complex systems [14], [15].

Definition 8. LFES Sequence-Independent Structural Degrees of Freedom [43]–[45]: The set of independent actions E_S that completely defines the available processes in a LFES. Their number is given by:

DOF_S = \sigma(E_S) = \sum_w \sum_v J_S(w, v) (9)
As has been shown in previous work [45], [62], [63], it is often useful to vectorize the LFES knowledge base. The shorthand \( [\cdot]^{V} \) is used to replace \( vec(\cdot) \). Furthermore, a projection operator may be introduced to project the vectorized knowledge base onto a one’s vector to eliminate sparsity. \( \mathbf{P} J_{\mathcal{S}}^{V} = 1^\sigma(\mathcal{E}_{\mathcal{S}}) \). While solutions for \( \mathbf{P} \) are not unique, this work chooses:

\[
\mathbf{P} = \left[ e_{\psi_{1}}^{\sigma(\mathcal{E}_{\mathcal{S}})}, \ldots, e_{\psi_{c}}^{\sigma(\mathcal{E}_{\mathcal{S}})} \right]
\]  

(10)

where \( e_{\psi_{c}}^{\sigma(\mathcal{E}_{\mathcal{S}})} \) is the \( \psi_{c}^{th} \) elementary row vector corresponding to the first up to the last structural degree of freedom in increasing order.

### III. TRANSPORTATION ELECTRICITY NEXUS HYBRID DYNAMIC MODEL DEVELOPMENT

This section describes the development of a hybrid dynamic model of the transportation electricity nexus on the basis of an earlier reported version [14], [15]. The development proceeds as follows. First, the TEN is considered as a special case of an LFES and consequently the TEN knowledge base is constructed (Section III-A). Special attention is given to the coupling between the transportation and power grids. This knowledge base is then used to construct a timed Petri-net model of the electrified transportation system’s discrete-event dynamics (Section III-B). The continuous time evolution of kinematic and electrical states in the electrified transportation system and the electrical power grid are then added between discrete event states (Section III-C). It is important to note that this model describes the physical dynamics of the TEN as the evolution of discrete and continuous states. It does not propose decisions or control algorithms that drive the model’s inputs.

#### A. Transportation Electricity Nexus Knowledge Base

The transportation electricity nexus knowledge base \( J_{TEN} \) is constructed from two smaller knowledge bases; one for the electrified transportation system \( J_{ETS} \) and one for the electric power system \( J_{EPS} \). Each of these is a specialized LFES in its own right. Therefore, to guide the discussion Table II summarizes the specialization of LFES concepts to a TEN.

1) **Electrified Transportation System Knowledge Base:** The construction of the electrified transportation system knowledge base is done in two steps; first neglecting and then later considering electrification. Neglecting electrification, transportation systems have buffers. Those capable of allowing the entry or exit of travelers (e.g. stations, parking lots) may be considered transforming resources \( M_{ETS} \) in that they change the logical state of the traveler while those not capable (e.g. intersections) are considered independent buffers \( B_{ETS} \) [45].

The transporting resources \( H_{ETS} \) include roads as well as rail, bus, and metro lines [45]. Any of these system resources \( R_{ETS} \) may be logically organized by Equation 1 into several modes of transport \( R_{ETS} \). Collectively, they realize the entry/exit processes \( P_{\eta ETS} \) as well as the transportation processes \( P_{E ETS} \) [45]. Here, it is important to note that the inclusion of the \( \sigma(B) \) parking processes mentioned in Definition 5 is essential to the TEN model. Many traffic simulation packages, perhaps due to their historical focus on road congestion management, define vehicle trips between distinct origins and destinations [38], [39]. The parking processes, at best, have little to add in that regard and can be left out of the scope of simulation. At worst, they dramatically expand the number of vehicles participating in the evolution of the traffic system’s state. In a transportation electricity nexus, however, many of these parking functions are also associated with charging, and thus can not be neglected. Thus, transportation-electricity nexus modeling must be done from the perspective of the traveler and thus is consonant with ongoing multi-agent system trends in future urban transportation systems [67], [68]. These system processes and resources can be considered collectively exhaustive and mutually exclusive for transportation systems neglecting electrification.

In the transportation electricity nexus, however, this assumption must be relaxed to account for charging functionality. Therefore, a set of charging processes are introduced as a specialization of the holding processes found generally in LFESs.

#### Definition 9. Charging Process [14], [15]: A resource-independent process \( p_{\gamma} \in P_{\gamma} \) that positively or negatively affects an electric vehicle’s state of charge (SOC). These processes may draw or inject the required energy into the interdependent electricity grid.

In the context of this work, \( P_{C} = \{ p_{c1}, \ldots, p_{c4} \} \) [15] where:

- \( p_{c1} \) – null charging does not change the electric vehicles state of charge
- \( p_{c2} \) – discharge the EV SOC to the electric vehicle’s propulsion system
- \( p_{c3} \) – charge the EV SOC by wire
- \( p_{c4} \) – charge the EV SOC wirelessly

At this high level of design, no assumption is made on sign (or directionality) of the power transfer. These processes may be realized by the set of resources \( R_{ETS} \). Conventional (non-electrified) stations effectively implement \( p_{c1} \) while conventional roads implement \( p_{c2} \). Charging stations and electrified roads are capable of \( p_{c3} \) regardless of whether they are simply charging or implementing more advanced “vehicle to grid” technology [28]–[30]. Finally, the electrified roads associated with online electric vehicles [69]–[72] are capable of \( p_{c4} \).

The electrified transportation system knowledge base \( J_{ETS} \)
thus follows straightforwardly from Equations 7 and 8.

\[
J_{ETS} = \begin{bmatrix}
J_{METS} & 0 \\
J_{HETS} & 0
\end{bmatrix}
\] (11)

where \(J_{HETS}\)

\[
= \begin{bmatrix}
J_{PETS} \otimes 1^{\sigma(P_{ETS})} & 0
\end{bmatrix} \cdot \begin{bmatrix}
1^{\sigma(P_{ETS})} \otimes J_{HETS}
\end{bmatrix}
\] (12)

Interestingly, if \(P_{ETS}\) is taken as the null set, then \(J_{METS}\) vanishes, and the model describes a closed electrified transportation system where passengers are simply moved from one buffer to another. \(J_{HETS}\) describes the structure of the transportation system while \(J_{PETS}\) describes the structure of the charging system. In all, the electrified transportation system knowledge base is able to distinguish a collectively exhaustive and mutually exclusive set of processes out of the feasible combinations of transportation and charging processes. For example, these can include “staying in place at \(b_{y1}\) while charging by wire” or “moving from \(b_{y1}\) to \(b_{y2}\) while charging wirelessly”. A fully worked example of the electrified transportation system knowledge base is found in [14].

2) Electric Power System Knowledge Base: The construction of the electric power system knowledge base follows similarly (without consideration of the transportation system). A power grid’s transforming resources \(M_{EPS}\) are its generators and loads while its independent buffers \(B_{EPS}\) are its storage. Substations may be considered as storage of zero capacity. The transporting resources \(H_{EPS}\) are the power lines. Any of these system resources \(R_{EPS}\) may be logically aggregated by Equation 1 into several power grid areas \(R_{EPS}\). Collectively, they realize the generation/consumption processes \(P_{muEPS}\) as well as the transportation processes \(P_{nuEPS}\). Holding processes \(P_{nuEPS}\) may be introduced as necessary to distinguish between voltage levels in the power grid [45], [64]. The electrified transportation system knowledge base \(J_{ETS}\) thus follows analogously from Equations 7 and 8.

\[
J_{EPS} = \begin{bmatrix}
J_{MEPS} & 0 \\
J_{HEPS} & 0
\end{bmatrix}
\] (13)

It is important to note that each transforming resource in the electric power grid is only able to realize one process. Generators inject power, loads withdraw it. Transportation lines have two processes; one for each direction. A fully worked example of the electric power system knowledge base is found in [64].

3) Coupling in the Transportation Electricity Nexus: Construction of the (full) transportation electricity nexus knowledge base must recognize the coupling between the electrified transportation system and the electric power grid. This coupling arises from the recognition that \(R_{ETS}\) and \(R_{EPS}\) are not mutually exclusive. Rather any resource \(r \in R_{ETS}\) may potentially inject or withdraw power from the grid depending on whether it is capable of charging processes \(P_{c3}\) or \(P_{c4}\). Therefore, the transforming resources in the transportation electricity nexus \(M_{TeN} = M_{ETS} \cup B_{ETS} \cup H_{ETS} \cup M_{EPS}\) where \(M_{EPS}\) is taken as the set of generators, and loads unrelated to transportation. Also note that while electrified roads and rail lines appear as transportation resources in the electrified transportation system, they appear as transforming resources in the larger nexus system. Similarly, the transformation processes in the transportation electricity nexus \(P_{nuTeN} = P_{ETS} \cup P_{muEPS}\). The transportation electricity nexus knowledge base follows straightforwardly.

\[
J_{TeN} = \begin{bmatrix}
J_{MTeN} & 0 \\
J_{HTeN} & 0
\end{bmatrix}
\] (14)

\[
= \begin{bmatrix}
J_{METS} & 0 & 0 & 0 \\
J_{HEPS} & 0 & 0 & 0
\end{bmatrix}
\] (15)

This knowledge base \(J_{TeN}\) is used as the underlying system structure for the rest of the hybrid dynamic model development.

B. Timed Petri-Net Model of the Electrified Transportation System

At a high level, the physical dynamics of the electrified transportation system can be described by a timed Petri-net [35]–[37] based upon its structural degrees of freedom [14], [15]. For the sake of simplicity, and without loss of generality, the remaining discussion considers a closed electrified transportation system with respect to travelers. \(P_{muETS} = \emptyset\). \(M_{ETS} = \emptyset\). In developing the timed Petri-net model, the emphasis is placed on maintaining an intuitive link to the physical reality and thus it is defined with the following physical meaning:

Definition 10. Electrified Transportation System Timed Petri Net [14], [15]: A timed Petri net where \(N = (B_{ETS}, E_{ETS}, M, W, Q_B, Q_E, D)\).

- \(B_{ETS}\) is the set of places. It represents transportation independent buffers (e.g. stations & intersections).
- \(E_{ETS}\) is the set of discrete events. It represents the structural degrees of freedom (as defined in the previous section).
- \(M\) is the set of arcs. It represents the logical relationship from the events to the places and from the places to the events.
- \(W : M \rightarrow \{0, 1\}\) is the weighting function on the arcs. \(w(p_{y}, t_{w}) = 1\) and \(w(p_{y}, t_{w}) = 1\iff J(\omega, v) = 1\).
- \(Q_B\) is the place marking vector of size \(\sigma(B_{ETS}) \times 1\). It represents the queue of vehicles at a given transportation station awaiting an event. No restrictions are placed on the nature of the vehicle be it a car, bus, or train.
- \(Q_E\) is the event marking vector of size \(\sigma(E_{ETS}) \times 1\). It represents the number of vehicles undergoing the events (e.g. parking, charging, moving from one place to another).
- \(D\) is the event durations vector of size \(\sigma(E_{ETS}) \times 1\). It represents the duration of time that each event requires for completion.

Note that the structural degrees of freedom in the electrified transportation system described in the previous section fix the size of \(E_{ETS}, Q_E\) and \(D\). It is also important to note that the physical meanings of \(Q_B\) and \(Q_E\) are subtly different.
from many other Petri-net models. In this model, parking is explicitly modeled as a transition with its associated timing, duration, & required capacity. The leftover tokens in $Q_B$ represent the queue of vehicles awaiting the next transition. In the case of conventional parking, this is assumed to occur immediately upon arrival provided that there is sufficient capacity. In the case of electric vehicles that have arrived to charging stations, the associated charging queues determine the electric vehicle fleet utilization.

The electrified transportation system knowledge base also serves to define the arcs between places and events. The timed Petri net incidence matrix $M$ is the sum of its positive and negative components $M_+$ and $M_-$.

$$M = M_+ - M_-$$  

which are straightforwardly derived using Equations 4 and 5.

$$M_{p-} = \sum_{y1=1}^{a(B)} e_{G1}^{(B)} \left[ P \left( \epsilon_{y1}^{(B)} \otimes I^e_{(B)} \otimes I^a_{(PC)} \otimes I^r_{(R)} \right) \right]^T$$

$$M_{p+} = \sum_{y2=1}^{a(B)} e_{G2}^{(B)} \left[ P \left( I^e_{(B)} \otimes \epsilon_{y2}^{(B)} \otimes I^a_{(PC)} \otimes I^r_{(R)} \right) \right]^T$$

The discrete-event dynamic evolution of the timed Petri-net is then described by a state space equation [15].

$$Q[k+1] = \Phi(Q[k], U^{-}_k, U^{+}_k)$$

where $Q = [Q_B; Q_E]$, and $U^{-}_k$ and $U^{+}_k$ are the input and output firing vectors. The initial value of $Q$ is determined from input data. Because timed Petri net have transitions with a finite duration, they must essentially be fired twice; once to mark the beginning of their execution with the input firing vectors, and once to mark their end with the output firing vectors. $\Phi$ is consequently expanded [15].

$$Q_B[k+1] = Q_B[k] + M_{x}^{k}U^{+}_k - M_{y}^{k}U^{-}_k$$

$$Q_E[k+1] = Q_E[k] - U^{-}_k + U^{+}_k$$

Note that unlike (generic) Petri-nets, timed Petri nets must also track the state of tokens within transitions $Q_E$. The input firing vectors $U^{-}_k$ are derived from the traffic demand data after routing which states [15]:

- when a given vehicle will begin a trip from origin to destination
- which route (i.e. sequence of roads) it will take.
- how long it will remain at its destination
- when it will charge along the way (while moving or parked)
- subject to the capacity constraints on all the events
- subject to road traffic and rail signaling

The capacity constraints vector $C_E$ applies to each of the structural degrees of freedom.

$$Q_E[k] \leq C_E$$  

From an implementation perspective, this work assumes several well-established transportation system engineering techniques have already been applied. Microscopic traffic simulations require data collection from a large number of heterogeneous sensors across a region [39]. These are used to develop offline a consistent set of origin-destination data [39]. At that point, this data must be detailed via a routing algorithm that considers capacity constraints and transportation system signaling so as to generate input firing vectors $U^{-}_k$. This approach is consistent with MATSIM [73] & SUMO [74]; two leading open-source microscopic traffic simulators [75]. Indeed, the timed Petri net provided here and its associated transition data is equivalent to these simulators when electrification functionality is neglected [75].

The input firing vectors are very much tied to the concept of multi-agency in microscopic traffic simulation. In this regard, it is necessary to track not just the number of vehicles moving from an origin to a destination but also track each vehicle $l \in L$ as it undergoes its transitions over the course of the day: be they moving between distinct origin and destinations or staying in place to park or charge. To that effect, the traffic demand data after routing is captured in a set of vehicle firing matrices $U_k$ [15].

**Definition 11. Vehicle Firing Matrix** [15]: a binary vehicle firing matrix $U_k$ of size $\sigma(S) \times \sigma(L)$ whose element $U_k(\psi, l) = 1$ when the $k^{th}$ firing timing triggers a vehicle $l$ to take structural degree of freedom $\psi$ for action.

The potential for multiple fleets of vehicles is also supported. It requires that the columns of $U_k$ be ordered to allow a vertical partitioning into several column blocks each representing a fleet. Consequently, the input firing vectors for the timed Petri net $U^{-}_k$ are easily calculated [15].

$$U^{-}_k = U_k 1^{\sigma(H)T}$$

Therefore, the input firing vector $U^{-}_k$ represents an exogenous quantity calculated from the vehicle firing matrix. In contrast, the output firing vector $U^{+}_k$ is calculated from the event durations $D$ by means of a scheduled event list.

**Definition 12. Scheduled Event List** [15], [35]: A tuple $S = (u_{vk}, t_k)$ consisting of all elements $u_{vk}$ in firing vectors $U^{-}_k \cup U^{+}_k$ and their associated times $t_k$. For every element, $u_{vk} \in U^{-}_k$, there exists another element $u_{vk}^{+} \in U^{+}_k$ which occurs at $t = t_k + d_v$.

Consequently, the output firing vectors $U^{+}_{Dk}$ are then calculated from their elements for all the unique times $t = t_k + d_v$.

**C. Inclusion of Continuous-Time Phenomena**

The development of the transportation-electricity nexus hybrid dynamic model now requires the inclusion of the continuous-time phenomenon; namely the evolution of kinematic and electrical states in the electrified transportation systems and the electrical power grid. These appear as continuous differential and algebraic equations over the time domain between the discrete events of the previous subsection. It is also important to note that while the previous subsection assumes that events in the previous section were of fixed duration $D$, this section relaxes this assumption and replaces
them with variable durations determined by the motion of each vehicle.

The development of the hybrid dynamic model gains its inspiration from hybrid automata [35]. To that effect, it is defined as follows:

**Definition 13.** Transportation Electricity Nexus Hybrid Dynamic Model: A 10-tuple \( \mathcal{H} = (B_{ETS}, \mathcal{E}_{ETS}, M, W, Q, \Phi, U, X, f, domain) \) where

- \( (B, \mathcal{E}, M, W, Q) \) is the underlying marked Petri net (Definition 10).
- \( \Phi \) is the discrete state Petri-net transition function (Equation 19).
- \( U \) is a binary vehicle firing matrix for all times \( k \) (Definition 11).
- \( X \) is a continuous-time state vector representing the kinematic and electric state of the transportation electricity nexus.
- \( f \) is a vector field. \( f : Q \times X \times U \rightarrow X \). It describes the continuous-time evolution of state vector \( X \).
- \( domain \) is a set of invariant conditions [35] which associates a discrete state \( Q \) to an interval of \( X \) and \( U \) within which \( X \) and \( U \) must remain in order to also remain in the discrete state \( Q \).

The state vector \( X = [X_T, X_E] \) is divided into a state vector for the electrified transportation system \( X_T \) and another for the electric power grid \( X_E \). The former consists of the state vectors \( x_{TI} \) for a given vehicle \( l \in L \). While this hybrid dynamic model can easily accommodate an elaborate description of the vehicle’s internal dynamics, it is important to recall that doing so might be practically infeasible given the sheer number of vehicles being simulated within the TEN. To that effect, a minimalist model is chosen. \( x_{TI} = [z_I, \hat{z}_I, s_I] \) where

- \( z_I \) is the distance of the vehicle along a road segment in relative coordinates.
- \( \hat{z}_I \) is the speed of the vehicle along the road segment.
- \( s_I \) is the vehicle’s state of charge.

The state vector of the electrified transportation system \( X_T \) is derived from its structural degrees of freedom and well-known electric power engineering models. In general, \( X_E \) consists of the electrical state vectors \( x_{E\psi} \) for each structural degree of freedom \( \epsilon_\psi \in \mathcal{E}_{TEN} \). Therefore, this hybrid dynamic model keeps track of the state for all electric power grid resources including the power lines. This may not be entirely necessary for many electric power grid studies (e.g. transient stability, small-signal stability, frequency stability, power flow analysis). However, the model should allow the potential for power lines to have state as is the case of advanced control concepts (e.g. FACTS device integration). For the sake of simplicity, as a degenerate case, a state suitable for power flow analysis is chosen. For all power grid buffers \( B_{STEN} = M_{TEN} \cup B_{EPS} \), \( x_{E\psi} = [P_\psi, Q_\psi, v_\psi, \theta_\psi] \) where

- \( P_\psi \) is active power injected by the electric power grid buffer.
- \( Q_\psi \) is the associated reactive power.
- \( v_\psi \) is the associated voltage magnitude.
- \( \theta_\psi \) is the associated voltage angle.

The typical taxonomy of PV, PQ, and reference resources (i.e. buses) are assumed for \( M_{EPS} \cup B_{EPS} \) with their associated given values. The electrified transportation system resources \( R_{ETS} \) are assumed to be PQ resources with zero reactive power. Their active powers are given by:

\[
P(\psi) = \alpha(\psi)Q_E(\psi) \quad \forall \psi \in \mathcal{E}_{ETS}
\]

where \( \alpha \) is a vector of size \( \sigma(\mathcal{E}) \times 1 \) whose elements give the charging rate in kilowatts for the given event.

The choice of power flow analysis necessitates that \( f \) be divided into a set of differential equations \( f_t \) and a set of algebraic equations \( f_e \). \( f_e \) and \( f_e \) are the well known power flow analysis equations.

\[
P + jQ = \text{diag} (V) Y V^* \tag{25}
\]

where \( P_E = [P_{E\psi_1} \ldots P_{E\psi_n}]^T \), \( Q = [Q_{\psi_1} \ldots Q_{\psi_n}]^T \), \( V = [v_{\psi_1} \ldots v_{\psi_n} \ldots v_{\psi_n}]^T \) and \( Y \) is the bus admittance matrix. Here \( n = \sigma(B_{STEN}) \); the number of structural degrees of freedom with the power grid buffers.

The differential equations of the transportation system \( f_t \) are implemented in state space form:

\[
\dot{X} = f_T(Q, X, U_k) \tag{26}
\]

In free driving conditions, the dynamics of each vehicle become entirely uncoupled and the state of the vehicle becomes purely a function of the vehicle firing matrix.

\[
\begin{bmatrix}
\dot{z}_l \\
\dot{\hat{z}}_l \\
\dot{s}_l
\end{bmatrix} =
\begin{bmatrix}
\beta(\psi) \\
0 \\
\alpha(\psi)
\end{bmatrix}U_k(\psi, l) \tag{27}
\]

The vehicle speed is set to a constant speed \( \beta(\psi) \) whether it is moving along a road or parked at a station. Additionally, the charging rate \( \alpha(\psi) \) is sufficient to describe all four types of charging processes. One advantage of the free-driving model is that it retains the timings of the underlying timed Petri-net model. In contrast, under more normal driving conditions with some congestion a car following model is typically used [39], [76], [77]. Consequently, the state of charge is often modeled to change with the vehicle speed and vehicle firing matrix.

\[
\begin{bmatrix}
\dot{z}_l = z_l \\
\dot{\hat{z}}_l = \alpha_v \beta_v(t)(\hat{z}_{l-1}(t-T)-z_l(t-T)) \\
\dot{s}_l = f(\hat{z}_l) + \alpha U_{kl}
\end{bmatrix} \tag{28}
\]

where \( \dot{z}_l, \dot{\hat{z}}_l, z_l \) are the acceleration, speed and position of the \( l^{th} \) vehicle which follows the \( l-1 \) vehicle. \( \alpha_v > 0 \), \( \beta_v \) and \( \gamma \) are model parameters that control the proportionalities and \( T \) is reaction time [76], [77].

Finally, the **domain** describes a set of invariant conditions upon which a given discrete state remains valid [35]. In the context of the HDM, these conditions are useful for constraining the vehicles distance along the road segment and its state of charge within limits. For example,

\[
0 \leq z_h \leq z_{max} \\
0 \leq s_h \leq S_{max} \tag{29}
\]

where \( z_{max} \) may be the road length and \( S_{max} \) may be the vehicle’s battery capacity.
While the timed Petri-net model is sufficient to describe many aspects of the TEN, it is important to note that the inclusion of the continuous-time dynamics serves several practical purposes. The primary impact is to shift the timing of the discrete event states. A secondary impact is that the clumping of vehicles into high density waves of vehicles affects the vehicle queuing behavior. Finally, the degree to which the vehicles require additional charging from quickly depleted battery charge depends on the aggressivity of the driving behavior [3]–[5]. In all, the inclusion of the continuous time behavior allows the incorporation of the power grid phenomena and the intra and inter-vehicle dynamics to modeling resolution required for analysis. The insertion of these continuous phenomena within a timed Petri-net facilitates holistic performance analysis using results from the discrete-event systems literature [35]–[37].

IV. TRANSPORTATION-ELECTRIFICATION TEST CASE

With a hybrid dynamic model for the transportation electricity nexus in place, the paper shifts focus to its application in a test case. Here, the “Symmetrica” test case [15], [78] is chosen. A full rationale for the methodological use of test cases for the TEN application has been previously reported [78]. As shown in Figure 1, it consists of interlinked electrified transportation system and electric power grid topologies. The transportation topology consists of a 12x12km grid with intersections at every kilometer. Five charging stations are placed in the city center at coordinates (4,4), (4,8), (8,4), (8,8) and (6,6). Additionally, the peripheral nodes represent home charging. Finally, 13 groups of 2km x 2km electrified road crosses are uniformly distributed across the area. The power grid topology is a modified version of the 201-bus distribution system test case [79], [80]. The hetero-functional traffic demand consists of 6086 vehicles classified on the basis of their physical dynamics. This includes internal combustion, online-electric and plug-in electric vehicles in a ratio 2:1:1. Its behavior represents a home-commute-work-commute-home use case as a simplification of an average work day. Vehicles enter from the peripheral nodes, go to five work locations coinciding with the charging stations and then after 8 hours return back to periphery. The routes chosen for the two daily commutes are evenly distributed. The frequency with which vehicles begin the morning commute is exponentially distributed around 8am. This is asymmetrical because electric vehicles either begin or await charging upon arrival to the workplace and therefore do not count as “non-electrified” parking. Similarly, the number of parked vehicles at home rises sharply after the peak congestion times of 4pm and eventually stabilizes after charging. Also note that the numerical scale of parked vehicles is very much greater than those in motion. Here, the peak is just over 1000 vehicles – two orders of magnitude greater. From a systems perspective, roads are associated with distributed quantities while parking lots are associated with centralized ones. While this is perhaps a straightforward insight in transportation behavior, the implications on electrification infrastructure appear profoundly in the subsequent plots.

In Figure 2a, the number of driving vehicles on a given road segment is shown with a distinct color. In the hybrid dynamic system model, this corresponds to the rows of $Q_E$ that are associated with events along road segments. As mentioned in the previous section, the traffic demand has two exponentially distributed peaks of traffic congestion corresponding to the morning and afternoon commutes. The similarity in the shape of these two distributions suggest that the 8 hours devoted to work were sufficient to allow all electric vehicles to depart without delay. Therefore, both electrification scenarios demonstrate a 100% quality of service [11]–[13] for this specific use case.

In 2b, the number of parked vehicles at any given station is shown and corresponds to the rows of $Q_E$ that are associated with parking events. As expected, the number of parked vehicles at work locations rises sharply shortly after the peak congestion times of 8am and eventually stabilizes as moving vehicles reach their workplace. The shape of this phenomena is asymmetrical because electric vehicles either begin or await charging upon arrival to the workplace and therefore do not count as “non-electrified” parking. Similarly, the number of parked vehicles at home rises shortly after the peak congestion times of 4pm and eventually stabilizes after charging. Also note that the numerical scale of parked vehicles is very much greater than those in motion. Here, the peak is just over 1000 vehicles – two orders of magnitude greater. From a systems perspective, roads are associated with distributed quantities while parking lots are associated with centralized ones. While this is perhaps a straightforward insight in transportation behavior, the implications on electrification infrastructure appear profoundly in the subsequent plots.

In Figure 2c, the quantity of stationary charging vehicles is shown and corresponds to the rows of $Q_E$ that are associated with charging events; be they on electrified roads or at charging stations. Thus, the results demonstrate a superposition of behaviors that mirror the results of the fifth subplot; one that closely follows the moving traffic fleet and another that closely follows the parking of conventional vehicles.

In Figure 2d, the quantity of queued vehicles is shown and corresponds to $Q_B$ in the model. It exhibits two phenomena. In the first, queues of vehicles at homes are lengthy but shorten over the course of the morning commute hours until they disappear. Again, one feature of the model is to distinguish between vehicles that are “appropriately” parked (as in the second subplot) and thus appear in $Q_E$ and those that are queued to park and thus appear in $Q_B$. This distinction is emphasized (albeit arbitrarily) with the former describing the vehicles arriving home and the latter describing the vehicles about to leave in the morning. The second phenomenon is a queue at work associated with an under-capacity of conventional charging stations. This distinction is fundamentally important in the sizing of not just stationary charging stations but also centralized parking lots. Note here, that during works hours, the relatively centralized state of the vehicle fleet means that more than 100 plugin electric vehicles can be waiting to charge at time. Furthermore, the queue does not clear for
several hours. Had this queue been longer, with perhaps an increase in the traffic demand or electric vehicle penetration rate, it could have meant that some plugin electric vehicles would not be available on time for the afternoon commute home. This would represent a reduced quality of service. Alternatively, had this electrification scenario been applied to a taxi use case then many taxis would have to wait several hours after their morning customer just to take on another fare. Such a low vehicle utilization rate could prove to be a non-starter for EV taxi adoption. In contrast, the online electric vehicles could continue to operate in a seemingly perpetual way.

In Figure 2e, the online and stationary charging loads are
shown and correspond to Equation 24. It is here that the differences between the two electrification concepts becomes most apparent. Online charging occurs literally “on-the-go”, as required, in an entirely unmitigated way, as a result of road congestion. Thus, two sharp charging peaks colored in red emerge. In contrast, the conventional charging load depicted in blue is associated with a “stock” of vehicles and not their flow. For the charging at work, the number of charging vehicles climbs rapidly shortly after the peak congestion and saturates due to limited charging capacity. The charging load eventually falls in a step-wise fashion as the queues are cleared in the late morning hours. For home charging, the vehicle fleet is very much distributed. As a result, the charging load does not saturate and more closely resembles the sharp peak of online electric vehicles. Managing the magnitude and shape of these charging load profiles is of ultimate importance for successful integration with future electricity grids.

Finally, in Figure 2f, bus voltage magnitudes are shown upon completion of a power flow analysis at each discrete event step. More specifically, the voltages of Buses 30, 31, 40, 44, 139, 148, and 173 are shown. These correspond to the terminals associated with the longest branches of Symmerica’s radial power distribution system topology. The effects of the charging loads on the bus voltages coincide with the two main peaks at approximately 8am and 4pm. The morning bus voltages, in this case, are minimally depressed to approximately 0.99 pu. Note here, that many of the morning’s charging facilities appear relatively close to the feeder location at Bus 201. In the meantime, the evening bus voltages are depressed much more substantially to as low as 0.96. While this is partially caused by the magnitude of the charging load, it is particularly amplified by the placement of several charging stations on the same feeder branch near its terminal. This raises questions as to the most appropriate nature of a power distribution system topology given a symmetrical and meshed electrified transportation system.

VI. CONCLUSION & FUTURE WORK

In conclusion, this paper has provided a hybrid dynamic system model for hetero-functional transportation electrification. It was developed with the recognition that electrified transportation effectively couples the transportation systems to the electrical power grid in a “transportation-electricity nexus”. The model is used within Axiomatic Design for Large Flexible Engineering Systems as a framework fundamentally well suited to addressing hetero-functionality in networked systems. This foundation served to construct a marked timed Petri-net model which was then superimposed on the continuous time kinematic and electrical state evolution of the combined system. This model was then simulated using a newly developed test case called Symmetrica. It featured a symmetrical and meshed electrified transportation system connected to a radial power distribution system. It also featured a traffic demand corresponding to a typical work day. The results showed the relative benefits of plug-in and online electric vehicles from both transportation and electric power system perspectives. The model also serves to address some of the deficiencies found in a recent review of existing software originally intended solely for microscopic traffic simulation [75] and meets many of the elements required to engineer a robust software solution [15].

In future work, this hybrid dynamic model for hetero-functional transportation electrification has the potential to be used in the context of Intelligent Transportation-Energy Systems (ITES) first mentioned in [12], [15], [78]. As shown in Table 1, the physical coupling of the transportation and power systems bring about coupled operations management decisions. Although the current model is currently meant for offline study, greater telemetry of the transportation-electricity nexus may open the possibility of online applications. Such telemetry would be enabled by recent developments in intelligent transportation infrastructure, connected vehicles, and smart (power) grids [81], [82]. These applications are potentially more viable in commercial uses cases such as taxis and fleet vehicles where such telemetry is already common. The Abu Dhabi Electric Vehicle Integration study [12] assumed an EV taxi integration scenario. Meanwhile, dynamic routing algorithms [47] have an established literature in routing fleets of commercial vehicles on the basis of their currently measured position.

Such decisions in an ITES may present many benefits. In conventional charging, an ITES can serve to modify vehicle dispatch, route choice and manage queues so as to minimize the waiting time for charging. Here, there is the potential that dynamic routing algorithms can use measured vehicle position to not just reduce congestion on roads but also congestion at charging facilities. This would lead to direct improvements in quality of service and set precedent for other EV integration scenarios with other use cases. Charging queue management, coordinated charging [16]–[27], and vehicle-2-grid stabilization [28]–[30] also serve to reshape the charging load profile on the power grid. In the case of online electric vehicles, quality of service has been effectively guaranteed. However, an ITES may still have the potential to modify vehicle dispatch, route choice, and implement coordinated charging and vehicle-2-grid stabilization to smoothen charging loads that will directly follow traffic congestion. The enhancement of this technical performance can also lead to direct cost savings in investment and operating costs [15]. Shifting the timing of charging peaks avoids the installation of peak load capacity. Meanwhile flattening the shape of charging loads means that fewer operating reserves will ultimately be required. An ITES, therefore, may present a valuable opportunity to improve the techno-economic case for a sustainable and electrified transportation system [15]. While the development of such decision-making techniques, especially with the telemetered data, presents formidable challenges, it is likely that it will be facilitated by recent trends in connected vehicle technology and multi-agent systems [81].

REFERENCES

Amro M. Farid received his Sc.B and Sc.M degrees from MIT and completed his Ph.D. degree at the Institute for Manufacturing within the University of Cambridge Engineering Department in 2007. He is currently an associate professor of engineering and leads the Laboratory for Intelligent Integrated Networks of Engineering Systems (LIINES) at the Thayer School of Engineering at Dartmouth, Hanover, NH, USA. He is also a research affiliate at the MIT Mechanical Engineering Department. His research interests address the systems engineering of intelligent energy systems including smart power grids, energy-water nexus, transportation electrification, and industrial production. He is a senior member of the IEEE and is actively involved in the Control Systems Society, the Systems, Man & Cybernetics Society, and the Industrial Electronics Society.